Case study micro-report

Using the Cambridge Mathematics Framework to refine the UNICEF-Cambridge Curriculum Progression Framework (Mathematics)

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Mapping a curriculum to a reference framework, creating and refining a curriculum and producing effective documentation are all key activities which we are designing the Cambridge Mathematics (CM) Framework to support. We worked with a team in the Applied Research Division in Cambridge Assessment, in partnership with UNICEF, to develop a mathematics curriculum for education in emergency (EiE) situations, serving learners who have been displaced and have no access to formal schooling. As part of this collaboration, we developed tools and processes for mapping, refining and documenting a curriculum framework which are applicable to a wider variety of mapping scenarios, including curriculum analysis, curriculum comparison and curriculum-assessment alignment. Our work in this case resulted in the completion of the mathematics curriculum framework for UNICEF's Learning Passport for Children on the Move (LPCM) programme and in new tools, processes and insights for the Cambridge Mathematics team which will contribute to the support we provide to users of the CM Framework.

Introduction

This micro-report presents a curriculum refinement case study featuring the inclusion of mathematics in the UNICEF-Cambridge Curriculum Progression Framework. We present the background for the series of case study micro-reports separately[1]. The results and discussion offer an example of what can be learned from refining a curriculum through alignment with the CM Framework.

Refining a newly designed curriculum is a common curriculum design goal. Activities in this case which contributed to this goal can be grouped into three phases:

1. Creating an initial curriculum framework for a specified context, drawing on existing curricula
2. Modifying the initial curriculum framework based on mapping to the CM Framework
3. Communicating the final curriculum and laying the groundwork for implementation support

In this report, we focus on the role of the CM Framework in phases 2 and 3. Phase 1 did not involve the CM Framework and will be reported separately by the UNICEF-Cambridge Curriculum Progression Frameworks design team.

The main processes we examine, with respect to the CM Framework, are *mapping*, *refining* and *documenting*. Mapping a curriculum to a reference framework and using the results of that mapping to refine and support enactment of the curriculum on the basis of that alignment are key anticipated uses

[1] see Background for the case study micro-report series (Jameson, 2019)

of the CM Framework. For the purposes of this case study, we define *mapping to the CM Framework* as creating a link which indicates a correspondence between an element of external mathematical content and an element of mathematical content within the CM Framework. We consider *curriculum refinement* in this case to be changes made to the composition and structure of the set of individual topics (which we refer to as *curriculum statements*) making up each subdomain of a mathematics curriculum. *Documenting a curriculum* in this case refers to ways of representing it which are useful for the people who need to enact and support it; e.g. teachers, planners, and resource developers.

Case context and goals: mapping to a reference framework and refining a curriculum

The mapping described here occurred as part of the development of a framework of mathematics learning sequences for the *Learning Passport for Children on the Move* (LPCM) programme; other components are science and literacy. These learning sequences are represented at a general level, but remain mapped to the more fine-grained content in the CM Framework. The purpose of the set of LPCM learning sequences is to provide guidance for those creating and delivering lessons and materials for use with learners from early primary to mid-secondary ages in Education in Emergency (EiE) situations – that is, learners who have been displaced as refugees or migrants and may not currently have access to formal schooling. Such learners vary widely in terms of the amount and structure of their education prior to displacement. A curriculum framework to help provide them with opportunities to learn coherently must be flexible and focused enough to be successfully implemented in a potentially disjointed manner by adults who may not be professional teachers.

Members of the CM Framework team worked with the Education and Curriculum Group in Cambridge Assessment's Research Division (ARD) to develop the LPCM Mathematics Framework (LPCMM). Based on prior curriculum research within ARD (Oates, 2017), the LPCMM Framework team identified key design principles which could inform the focus of their learning sequences (Johnson, Coleman, & Fitzsimons, 2019). Those most related to the work of the CM Framework team are:

- Flexibility: Teachers will need the structure and content of the LPCMM Framework to provide support for the range of learners' past experiences and the lack of materials or continuity which may affect teaching in EiE contexts.
- Parsimony: The idea that the LCPMM Framework should provide robust support for the fewest, most important topics necessary to provide learners with access to *powerful knowledge*. Reducing the number of topics compared to a formal, conventional curriculum will make it possible to teach more effectively while accommodating the range of learners' past experiences.

- Powerful knowledge: Knowledge which can be applied widely, which transcends learners' present experiences and "which many will not have access to at home, among their friends, or in the communities in which they live" (Young, Lambert, Roberts, & Roberts, 2014, p. 10). Access to powerful knowledge can give learners greater agency later on in how they may apply knowledge and skills from their education. Supporting this involves leveraging the connected nature of mathematical understanding.
- Coherence: The coherence with which students experience mathematics in school – within mathematics and between mathematics and other subjects – affects their opportunity to learn (Cunningham, 2017; Schmidt, Wang, & McKnight, 2005). This coherence can be improved by coordinating perspectives in curriculum design and teaching, and by aligning curriculum structure with existing evidence about student learning and conceptual connections in mathematics (Jameson, McClure, & Gould, 2018).

Both teams entered the joint development process with essential starting materials, including a first-draft curriculum. The goal of the LCPMM Framework team was to produce a refined version of their initial curriculum. They wanted it to be

- coherent,
- justified by research in mathematics education as well as initial observations of topics present across reference jurisdictions, and
- presented to planners and teachers on the ground in a way they could understand and act on appropriately.

The CM Framework team had the additional goal of developing tools and processes for curriculum mapping, curriculum design and curriculum documentation activities which could be applied more generally across a range of mapping scenarios. To inform this goal, we began the study with the following design questions (DQs):

(DQ 1) Starting with the CM Framework, what tools and processes were necessary?

(DQ 2) What steps required professional judgment and what information was needed at those points?

(DQ 3) How and to what extent could the process be automated and streamlined?

(DQ 4) Which formats would be most useful for documentation of the resulting curriculum?

(DQ 5) How could the process of creating this documentation be automated and streamlined?

Finally, we wanted to keep in mind the typical affordances of knowledge maps so that we could examine whether the tools and process we develop are yielding the benefits we might expect to gain from the use of the CM Framework. Based on a review of the knowledge mapping literature (Eppler, 2004), we would expect knowledge maps which are useful and usable in this type of case to support users in taking certain general kinds of actions, indicated in Table 1.

Table 1: Expected affordances of knowledge maps; all are relevant to some degree in this case

	Combined user groups	This case
Help designers to communicate ideas about knowledge to others; make tacit ideas explicit and present ideas in a form that users can relate to (Eppler, 2004; Vail, 1999)	✓	✓
Help users to “remember, comprehend, and relate knowledge domains through insightful visualization and aggregation of information” (Eppler, 2004, p. 200)	✓	✓
“[M]ake information actionable in new contexts, connect it with previous experiences” (Eppler, 2004, p. 189) – that is, professional learning and transfer	✓	✓
Help users to evaluate what knowledge is available for decision-making, and from what sources (Eppler, 2004)	✓	✓
Help users to see concepts within a bigger picture and to switch between multiple perspectives (Eppler, 2004)	✓	✓
Help users to evaluate and compare sets within knowledge domains – examining what knowledge is available, from what sources, and with what justification (Eppler, 2004)	✓	✓
Provide a “common framework” when searching for or contributing “relevant knowledge” (Eppler, 2004, p. 190), which itself supports professional learning	✓	✓
Contribute to the field by providing a big-picture perspective and a research base with respect to ideas that people in different roles may hold in common	✓	✓
Relate the big-picture perspective to different levels of underlying detail (Eppler, 2004)	✓	✓
Support professional learning in practical contexts: “just-in-time” (Vail, 1999, p. 23)	✓	✓

Role of this study in our evaluation plan

The LPCMM study has informed the initial development of tools and processes for mapping, curriculum refinement and documentation using the CM Framework. We will continue to build upon these tools and processes to refine the experience that curriculum developers have when they use the CM Framework.

Methods and materials

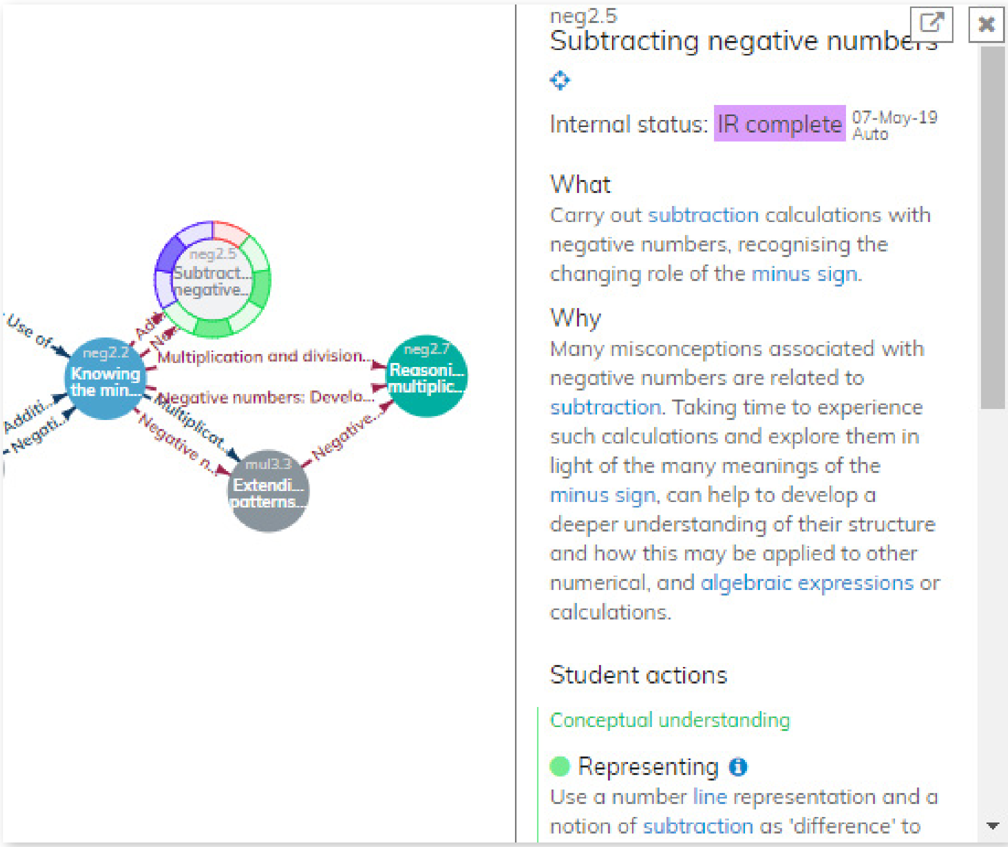
Cambridge Mathematics structure and content[2]

The CM Framework team worked with the network of mathematical experiences in the CM Framework. This network is our interpretation of the knowledge of students' learning which has been developed in the mathematics education community through empirical research and practice. In the CM Framework, mathematical ideas and key relationships between them are represented in a network of *waypoints*, which we define as "places where learners acquire knowledge, familiarity or expertise" (Jameson et al., 2019, p. 4). Waypoints have titles, descriptions and student actions, which are examples of the kinds of things students might do to help them build an understanding of the content at a waypoint. An example of a waypoint and its content is shown in Figure 1. We call the relationships between waypoints *themes*, and in the CM Framework they represent either the development of some part of an idea from one waypoint to the next, or the use of one idea contributing to the understanding of another[3].

Figure 1 on next page

[2] This subsection is reproduced from Case study micro-report: Mapping MathemaTIC tasks to the Cambridge Mathematics Framework (Jameson, 2019)
[3] Waypoints, themes, and types of student actions are described in Ontology: Structure and meaning in the Cambridge Mathematics Framework (Jameson et al., 2019)

Figure 1: Example waypoint with content ("what," "why" and "student actions")



Research Summaries are documents in which a subset of this network of waypoints and relationships is embedded; they "tell the story" of the content and structure of a set of waypoints and themes. Each Research Summary includes three potential elements of interest for this case:

1. a review of the literature informing the content,
2. the embedded CM Framework content and
3. a description of how our interpretation of the sources in our research base has led to the structure shown in that Research Summary.

We determined that Research Summaries would be the most accessible way, at our current stage of interface development, to present the CM Framework content for external use.

Research Summaries can be viewed as static documents or through the online CM Framework platform, CMF Nexus (Stevens et al., 2019). In CMF Nexus, the embedded content can be viewed dynamically so that the content for each waypoint can be viewed in full one waypoint at a time, or it can be output as a spreadsheet for users to scan across waypoint content in a different way.

Initial LCPMM Framework curriculum statements

The LCPMM Framework team formed an initial progression of topics, which we call *curriculum statements*, drawn from existing topics found in common across curricula in selected jurisdictions. Their methods, reference jurisdictions and initial curriculum statements have been reported in full by the LCPMM Framework team[4]. This work set the initial scope of the LCPMM Framework, with the intent of adding, removing, modifying and/or re-ordering them if a need for this was indicated by the CM Framework. The initial curriculum statements consisted simply of a short statement describing each topic; no additional information was incorporated, but the team which composed and ordered the initial statements brought their interpretation of the statements into the discussion.

Team backgrounds

On the LCPMM Framework team, criteria and methods for developing the initial list of curriculum statements consisting of mathematics were developed by researchers in the Assessment Research and Development (ARD) within Cambridge Assessment, whose backgrounds include research in mathematics education. Specialists in mathematics education, whose backgrounds included mathematics teaching and curriculum design, reviewed curriculum documents for selected jurisdictions and selected initial curriculum statements according to the agreed criteria.

CM Framework team members mapped content they had created for the CM Framework to seven domains: *Geometry*, *Measure*, *Algebraic working and thinking*, *Whole number and operations*, *Fractions*, *decimals and percentages* and four operations, Data handling, and Risk. Collectively, their backgrounds include multiple...etc. The CM Framework software developer, whose background includes mathematics and educational design, worked with them to make interface adjustments and new tools to facilitate curriculum mapping, refinement and documentation. A researcher on the CM Framework design team, with a background in educational design research in STEM, created a protocol protocol for diarising mapping activities and feature development requests.

[4] Details...(2019) and Oates, Fitzsimons, Coleman and Greatorex (2020); a full range of reports can be found here

Collaboration in curriculum design

The CM Framework team met weekly with the LCPMM Framework team (which in turn regularly checked in with UNICEF) to discuss the ongoing mapping and curriculum refinement. This helped them to get a sense check and adjust when necessary, so that the final result took relevant stakeholder perspectives into account. Later meetings provided opportunities to get a sense of the questions remaining at the end of Cambridge Mathematics' involvement in the project, and to be able to ask any questions necessary for reporting on the design process.

The pilot case protocol

The CM Framework team developed and trialled processes and tools for curriculum mapping and development. They kept meeting notes and diarised their work and the needs for specific new tools which grew out of it. At various points they presented summaries drawn from the design diary to internal and external audiences. The data we discuss below comprise the user self-report design diary along with meeting notes and summary presentations, snapshots of work in progress, final curriculum documentation, and notes from debriefing conversations comprise the data we discuss below.

Time frame

The CM Framework team spent around 140 hours to create the entire LCPMM Framework, from the mapping of the original curriculum statements to the final ordering and documentation of the refined curriculum statements. This included time spent in collaboration meetings. This work occurred over a period of eight months, woven into the CM Framework writing schedule, and involved curriculum statements covering the entire domain of mathematics. Early on, some of this time was spent manually doing steps which were later automated, so this time frame is not indicative of how long future mapping projects might take.

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Results

For the LCPMM Framework team, the result was the completed LCPMM Framework: a connected and conceptually coherent network of mathematics curriculum statements, with additional documentation in the form of tables to provide a useful overview by subdomain. Each statement was ordered according to the "earliest" that it could be addressed, relative to statements preceding and following it. The structure and content of waypoints mapped in from the underlying CM Framework, along with the Research Summaries explaining them, provided explicit justification for the connections between curriculum statements.

For the CM Framework team, the results were a set of processes for curriculum mapping, design and documentation and the addition of new tools for mapping to the CMF Nexus platform. These processes are summarised in Table 2. Steps 1 and 2 are processes for mapping, steps 3-7 for curriculum design and steps 8 and 9 for curriculum documentation. Each of these steps is distilled from the documents produced from the diarising and summarising, and from meeting notes and conversations within the CM Framework team.

Table 2: Steps in the current process of mapping, refining and documenting curriculum statements (CSs) (indicates professional judgement required; indicates automation)*

Step	Inputs	Actions	Outputs
1. Enter initial CSs into CMF Nexus	Initial CS spreadsheet	Upload spreadsheet	Initial ordered but unconnected CS nodes (Fig. 2, Appendix A)
2. Create initial simple subdomain map	Initial CS nodes	Map initial CSs to most relevant* waypoints in CM Framework, auto-summarise connections between waypoints	Initial simple subdomain map (Fig. 3, Appendix A)
3. Create full subdomain map	Initial simple subdomain map	Expand and combine subsets from connected waypoints	Full subdomain map (Fig. 4, Appendix A)
4. Create suggested waypoints map	Full subdomain map	Remove waypoints already mapped elsewhere or out of scope*	Suggested map of waypoints underlying initial CSs (Fig. 5, Appendix A)
5. Create suggested subdomain map	Map of suggested waypoints, initial CSs	Modify, reorder initial CSs, create and map in new CSs as needed*; label CSs with subdomain	Suggested subdomain map (Fig. 6, Appendix A)

Step	Inputs	Actions	Outputs
6. Create CS subdomain network	Suggested subdomain map for each subdomain	Order CSs within each sub-domain and auto-link connections between them via underlying waypoints	Network of subdomain CSs (Fig. 7, Appendix A)
7. Combine CS subdomain networks	CS networks for each subdomain	Combine CS subdomain networks; order and auto-summarise connections between subdomains	Full final CS network for the domain (Fig. 8, Appendix A)
8. Represent full CS network in tables	Full CS network for the domain	Show CSs in table by subdomain	Table of final CSs showing left-to-right progression in each subdomain (Fig. 9, Appendix A)
9. Create final list of top-level statements	Table of final CSs	Create high-level statements to categorise final CSs*	Final list of top-level statements to summarise the curriculum (Fig. 10, Appendix A)
10. Create complete high-level overview	Final list of top-level statements to summarise the curriculum	Create a table summarising content sequences in a small number of columns, organised by subdomain, so structure and content are viewable at a glance	Summary table of entire mathematics curriculum framework (Fig. 11, Appendix A)

Of these actions, auto-summarising and auto-linking take the most advantage of the graph data format we used. In order to auto-summarise a visualisation of a path between nodes, an algorithm finds and prioritises relevant paths and shows each path as a single edge labelled with the number of nodes in the path. Paths between any nodes whether waypoints or curriculum statements can be auto-summarised this way. Auto-linking allows curriculum designers to make the virtual links from auto-summarising into real, persistent links.

Steps 8 and 9 resulted from experimentation with several possible formats for documenting the final LCPMM Framework. We settled on the compact tables and lists mentioned in Table 2 because they were condensed enough to provide a big-picture overview at a glance for stakeholders engaging and managing curriculum development and implementation at a high level. The map of connected CSs, and the map of the waypoints and Research Summaries underlying the CSs, could provide further detail for those developing resources, assessments, schemes of work, professional development activities or lessons; these stages of curriculum implementation have not yet taken place.

Some actions in Table 2 involved developing a new tool, feature or practice in CMF Nexus. The CM Framework team worked to outline the requirements for each as they tried out different ways of working. The final set of new features added is shown in Table 3. The automating features in steps 3, 6 and 8 leveraged the graph database working environment to full advantage. They drastically reduced the amount of time and likelihood of sorting or scanning errors involved in these steps compared with performing the same steps manually in CMF Nexus or in spreadsheets.

Table 3: Adaptations and additions to CMF Nexus which emerged from mapping activities

Step	New tool, functionality or practice
1. Enter initial CSs into CMF Nexus	• Determined a set format for initial CS input • Created bulk upload and individual entry and editing forms • Added CSs to CMF Nexus search functionality
2. Create initial simple subdomain map	• Added ability for map visualisation window to display CSs • Added ability to create edges (connections) linking CSs to waypoints • Added ability to view the details of selected waypoints in a map all together in a table
3. Create full subdomain map	N/A
4. Create suggested waypoints map	N/A
5. Create suggested subdomain map	• Added ability to auto--identify and highlight cycles (circular paths of waypoints) to be resolved by designer judgment • Added labelling ability to CS nodes
6. Create CS subdomain network	• Applied existing network path auto-summarising feature to CSs
7. Combine CS subdomain networks	N/A
8. Represent full CS network in tables	• Created tabular output formats drawing on ordering determined by the full CS network
9. Create final list of top-level statements	N/A
10. Create high-level overview	N/A

We can condense these observations into a list of core actions for mapping, curriculum design and documentation:

Mapping

- **Finding mapping targets** in the CM Framework
    - **Using curated sets** of potential mapping targets and/or existing hierarchies (e.g. Research Summaries)
    - **Searching**
- **Finding closest match**
    - **Looking back and forth between details** of a CS and details of potential targets in the CM Framework
- **Creating connections** between CSs and target waypoints in the CM Framework

Curriculum design

- **Expanding map** from key target waypoints to include all waypoints which link them together
- **Reducing map** to eliminate irrelevant waypoints/pathways
    - **Judging a waypoint's relevance** by the development theme linking it in, the topics of the Research Summaries it contributes to, and/or whether it is covered elsewhere
- **Changing CSs** based on the map of suggested waypoints
    - **Filling in gaps** (adding new CSs to highlight important parts of the journey)
    - **Re-ordering** CSs
    - **Adding or changing subdomains**
- **Resolving conflicting implications for ordering** within a subdomain
    - **Identifying and resolving cycles** (circular paths of waypoints) so that a clear conceptual progression can be interpreted for the curriculum being created
    - **Case-by-case discussion** and agreement on priorities for ordering content which waypoints indicate could be prioritised equally in a conceptual progression
- **Shift focus from waypoints to CSs**
    - **Combine CSs into a network** based on connections between their underlying waypoints
    - **Use this network to decide on ordering** taking all subdomains into consideration

Curriculum documentation

- **Display the CS network as a table showing relative ordering** by subdomain
- **Display the CS network as a table showing absolute ordering** of CSs across subdomains
- **Display the high-level content summary statements in a table giving an overview of the entire curriculum framework at a glance**
- **List to/from connections** for each CS

In addition to these core actions, the CM Framework team noted that when creating the initial simple subdomain map for each subdomain (Step 2, Table 2), thinking in terms of clusters or communities of waypoints could be useful. Tools for cluster analysis are currently in development and could be a core action worth integrating in the future.

A principle guiding the processes of refining and ordering CSs also emerged from the collaboration and was applied to the process of creating the suggested subdomain map (see Table 2). This process was specific to the LCPMM Framework's intended use in EiE contexts. When finalising the reduced map and modifying, creating and reordering curriculum statements the teams collaboratively identified what they believed to be key ideas and actions in mathematics relative to the pool of content they were working with (Johnson et al., 2019). They decided to focus on identifying and including possible *threshold concepts* – those which open up "[a] new way of understanding, interpreting, or viewing something… they may be transformative…, irreversible…, and integrative…" and "may also be troublesome" (Meyer & Land, 2005, pp. 373-374). The team had characterised such concepts as being powerful in what they might enable students to do later on, and potentially likely to be powerful despite variation in cultural contexts (Rata, 2012, in Johnson et al., 2019). They agreed that these should appear at the earliest point possible according to the underlying conceptual dependencies, so that students would have as much opportunity to learn and build upon them as possible given the environment where their studies had been and perhaps would be further disrupted. This focus on key or threshold concepts provided useful guidance for reducing the waypoints map and ordering CSs.

Discussion

Through collaboration, we developed a set of tools and processes which allowed us to use the CM Framework successfully for curriculum mapping, development and documentation. We were able to determine which aspects of these processes could be automated and streamlined, and we identified key points where professional knowledge and judgment were required (see Table 2). Through trial and discussion we determined what forms of documentation could be passed along to stakeholders who require a high-level overview, and what forms of documentation (including the underlying waypoints map and Research Summaries in the CM Framework) might assist those involved in the details of implementing the curriculum, whether through creating resource materials, designing assessments, training teachers or planning lessons.

Working with CSs in the CM Framework provided many of the expected benefits of knowledge maps in this case (see Table 1). Some expected benefits, like professional learning, will need to emerge through implementing the CM Framework in programmes and partnerships. Similarly, until implementation we will not be able to see what happens when people in different roles coordinate with each other using the outputs of these processes. Other expected benefits, however, were fully realised. The different stages of curriculum design were made explicit through mapping and visualisation, and each stage of decision-making could therefore be explained and discussed more fully between designers, along with the justification behind decisions made at each step. This also made the resulting curriculum easier to justify to stakeholders. Switching between aggregated and detailed perspectives further aided design and discussion.

This case was most intended to inform the features of CMS Nexus which enabled the core mapping actions. Many of these features were already present within CMF Nexus framework design tools because they also enable core design actions, but this case helped us to be aware that we would also need to support them for end users engaged in mapping and curriculum design. The new features which made the biggest difference to the process were the ability to visualise a preliminary order for the full network of CSs based on their underlying waypoints (enabled by Steps 1 and 2 in Table 3), and the ability to create connections directly between CSs based on the connections in the underlying network of waypoints they were mapped to (Step 6, Table 2).

We can trace these core actions and features back to the design principles shaping the CM Framework and CMF Nexus and the theories behind those design principles. Doing this contributes to the overall logic model for our design, and allows us to keep track of the parts of our design which have been informed by each design principle and its accompanying theoretical influences. That way, we can see more explicitly how our design principles have shaped our design, and in cases where design features may not be enabling the user actions we expect, we can revisit our assumptions or even add new design principles to match unanticipated needs. Table 4 (Appendix B) shows the elements of our logic model which are involved in this case. Specific connections between elements are part of our discussions but are not shown here due to publishing constraints; a diagram showing all connections would need to be interactive to manage complexity.

We expect the list of core actions and features reported here to be applicable to other mapping contexts in which the CM Framework could potentially provide support. These include mapping for curriculum analysis, mapping for curriculum comparison, mapping to explore the alignment of assessments with curriculum, or even mapping other types of things like tasks to the CM Framework. Additional pilot cases will help us to explore what might be similar about these scenarios and what might require unique support in each.

We hope that our results might also be useful to other projects involving mapping for curriculum design, analysis or comparison. For example, the curriculum decision-making documentation format that the CM Framework team developed for reporting purposes in this case study could be used independently of the CM Framework in curriculum development scenarios where it is helpful for those not immediately involved to be aware of what decisions have been made intentionally and why. This could be useful when revisiting a curriculum for later revision, or in informing arguments among stakeholders.

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Appendix A: Curriculum development example: Area sub-domain

Figures 2 – 11 show examples of the curriculum development steps described in Table 2 and Table 3.

Figure 2: Step 1. Enter initial CSs into CMF Nexus; (a) shows a sample of the curriculum statements taken from the initial spreadsheet and (b) shows them as nodes in CMF Nexus

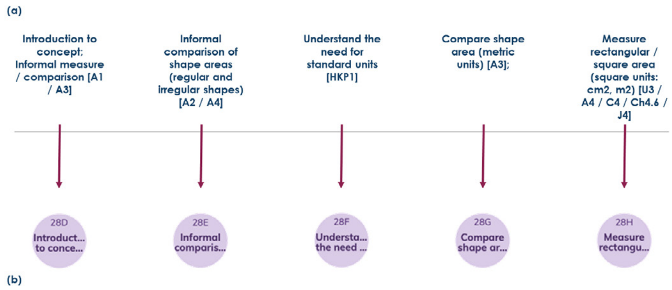

Figure 3: Step 2. Create initial simple subdomain map; the connections between waypoints are auto-summarised

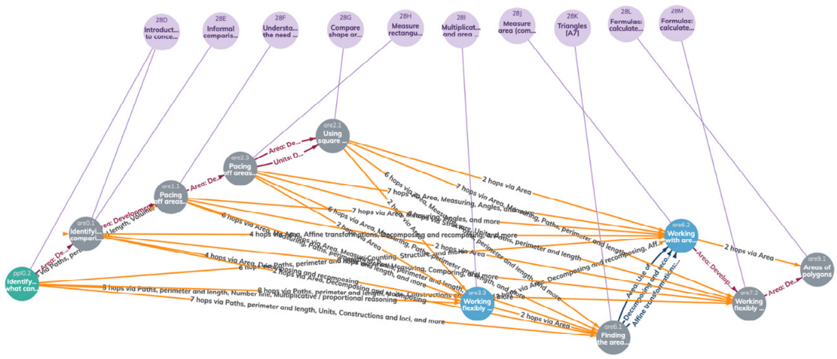

Figure 4: Step 3. Create full subdomain map; the map has been auto-expanded from the original set of waypoints

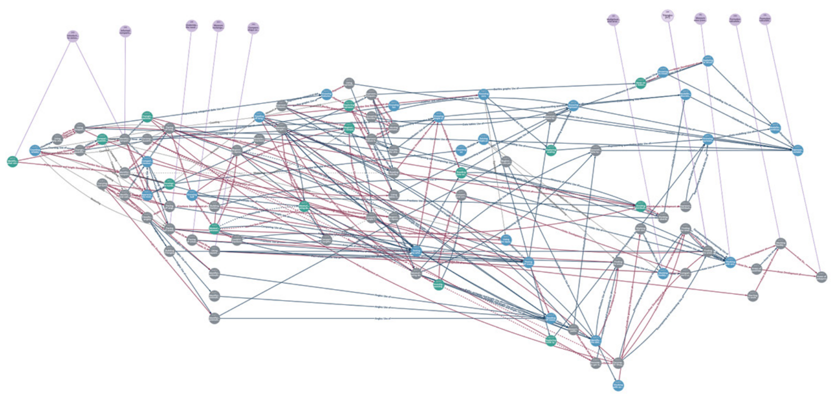

Figure 5: Step 4. Create suggested waypoints map; the expanded map has been manually reduced according to professional judgement about the requirements of this curriculum framework

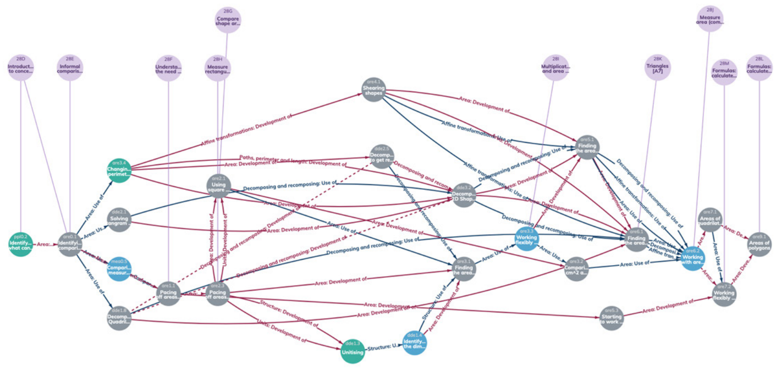

Figure 6: Step 5. Create suggested subdomain map; CSs have been manually added and/or re-ordered to best support the ideas represented in the waypoints

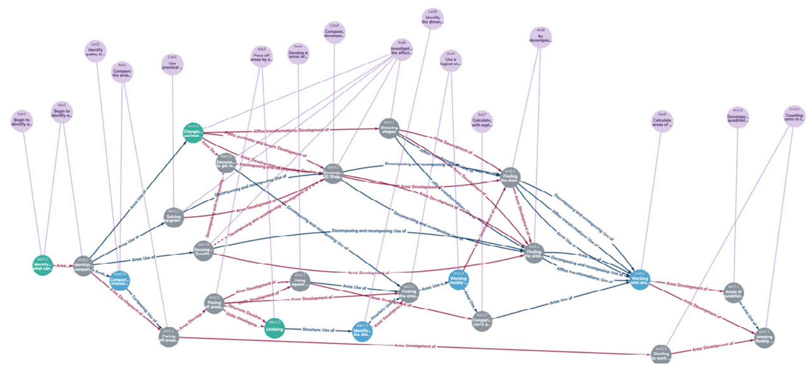

Figure 7: Step 6. Create CS subdomain network; connections between CSs is automated according to the underlying network of waypoints, but can be manually adjusted

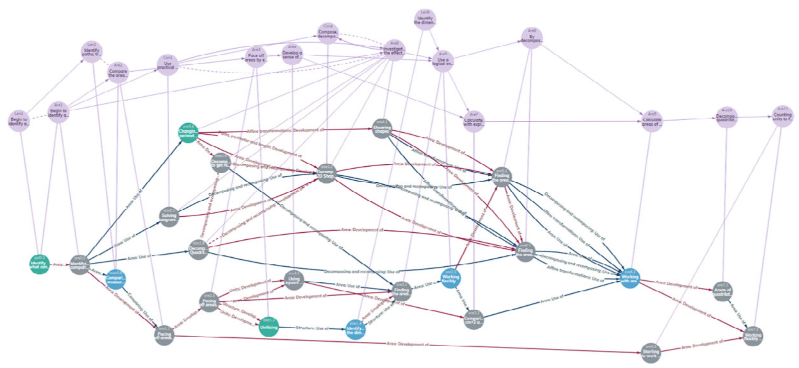

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Figure 8: Step 7. Combine CS subdomain networks (this map represents CSs in all of geometry)

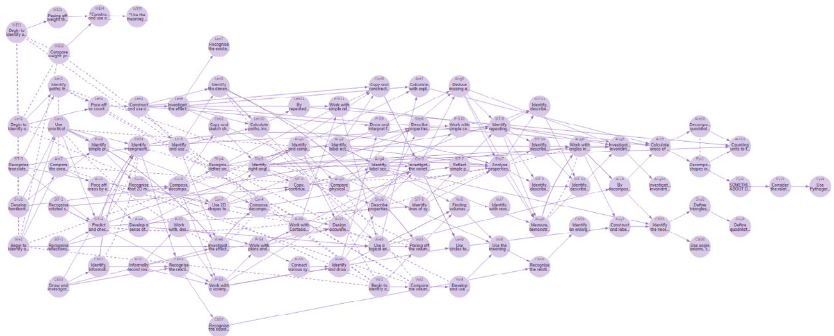


Figure 9: Step 8. Represent full CS network in tables (all of Geometry); the table is generated automatically from the titles, codes and connections of the CSs

WEI1 Begin to identify and compare the measurable attributes of objects. Compare weight: WEI3	WEI3 Pacing off weight through using an interim measure and a balance scale. WEI1 WEI4	WEI4 "Construct and use a measuring scale using a spring and non-standard unit. Develop WEI2, WEI3 WEI5	WEI5 "Use the meaning of kilo, hect, dec, deci, centi and mili to convert between metric units of mass WEI4	Len6 Investigate the effect of zooming in on a ruler on the structure of the measuring tool. Use the Len4 Are7, Are9, ...
Len1 Begin to identify and compare the measurable attributes of objects. Identify, compare and Len2, Len3	WEI2 Compare weight: predict, justify and confirm practically whether one is heavier, WEI4	Len3 Pace off or count lengths, paths and perimeters Len1, Len2 Len4	Len4 Construct and use a measuring stick using a non-standard unit. Estimate and measure Len3 Con8, Len5,...	Len5 Identify and use the correct measuring tool, and therefore unit, to measure angle, length or Len4 Are6, Len1...
SIT-3 Recognise translated shapes and patterns with translational symmetry. C&S1, Len2...	Len2 Identify paths; tracings of the movement of a point, including straight and curved paths. Len1, SIT-3 Len3	Shp2 Identify simple properties of 2D and 3D shapes such as number of edges, equality of Con1, Shp1 Con5	C&S1 Identify congruent lengths and angles by overlaying. Identify congruent lengths and C&S2, C&S... C&S4, Shp...	Con6 Compose, decompose and recompose 3D objects and shapes from 3D shapes. Con5, Shp1 Ang1, Con2...
Shp1 Develop familiarity with and identify (as a whole) 2D and 3D shapes, without relying on Con1, Con6...	Con1 Use practical equipment to reproduce and produce pictures and mathematical shapes in Are1, Shp1 Are6, C&S1...	Are3 Pace off areas by enumerating units in an array. Develop a logical enumeration of units in Are2 Are4	Con5 Recognise that 2D material can be used to make 3D objects, some easier than others. Shp2 Con6, Con7	P-S2 Work with, describe and design plans of relative position for objects in view. P-S1 P-S3, P-S4
Are1 Begin to identify and compare the measurable attributes of objects. Identify areas. Compare Are2, Are6, ...	Are2 Compare the area a shape covers by 'pacing off', using an interim (arbitrary) measure. Are1 Are3	SIT-4 Predict and check practically the position of mirror lines/lines of symmetry and the order Con1, SIT-1... C&S1, Shp...	Are4 Develop a sense of square centimetres and metres as a unit of area. Estimate and find the Are3 Are5, Are7	C&S4 Recognise the relationship between the co-ordinates of an object and its image under C&S1, C&S... C&S7
	SIT-2 Recognise rotated shapes and shapes with	C&S2 Identify, informally, similarities and	P-S1 Informally record routes through the immediate	

Figure 10: Step 9. Create final list of top-level statements

Geometry overview

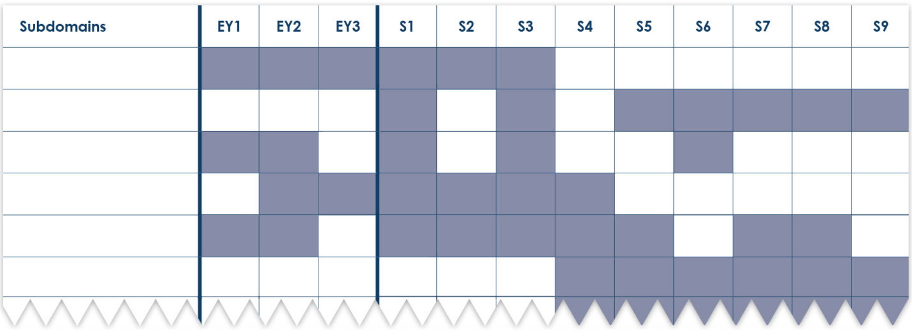
Shape

- Describe and recognise properties of 2D and 3D shapes using informal and formal language.
    - Number of sides, vertices
    - Number of edges, faces and vertices
    - Concerning parallel and perpendicular sides
    - Concerning parallel and perpendicular faces
    - Concerning symmetrical properties
    - Concerning equality of angles and side lengths
- Recognise necessary and sufficient conditions to define triangles and quadrilaterals. Use these to identify examples, non-examples and hierarchical family relationships.

Position and Space

- Record, communicate and follow informal representations for describing position and movement in the immediate environment.
- Read, communicate and follow information concerning describing position and movement in a variety of representations including plans, maps and simple compass directions.
- Work flexible with plans and elevations of 3D objects and mathematical shapes
- Work with Cartesian co-ordinates on a plane
- *Work flexibly with simple scaled floor plans.*

Figure 11: Step 10. Create complete high-level overview. The table was too large to show in full, so we illustrate its format with a diagram. Shaded boxes contain high-level summary content. Columns represent stages, with the LPCMM split into three early years stages and nine stages corresponding to primary and secondary



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Appendix B

Table 4: Looking from the design of the CM Framework to user actions in this case. See the Discussion section for an explanation of why it is useful to consider user actions, design principles and design features together

Design principles (specific)	Embodiment of design principles in CM Framework design features	User actions for curriculum mapping, design and documentation
• Connections • Fine-grained detail • Aggregate summaries • Searches/filters • Links to research and design justification	*Framework features and tools* • Research Summaries • Themes • Waypoint content • Query tool (waypoints, CSs, RSs, tags) • Tagging tool • Linking tool • Cycle identification tool *Framework content views* • Waypoint detail table view • Auto-summarised waypoint map view • Full waypoint map view • CS mapping view • Auto-summarised CS view • CS table view – compact • CS table view – spaced	*Mapping* • Finding mapping targets ◦ Using curated sets ◦ Searching • Finding closest match ◦ Looking back and forth between details • Creating connections *Curriculum design* • Expanding map • Reducing map ◦ Judging a waypoint's relevance • Changing CSs ◦ Filling in gaps ◦ Re-ordering ◦ Adding or changing subdomains • Resolving conflicting implications for ordering ◦ Identifying and resolving cycles ◦ Case-by-case discussion • Shift focus from waypoints to CSs ◦ Combine CSs into a network ◦ Use this network to decide on ordering *Curriculum documentation* • Display the CS network as a table showing relative ordering • Display the CS network as a table showing absolute ordering • List to/from connections

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