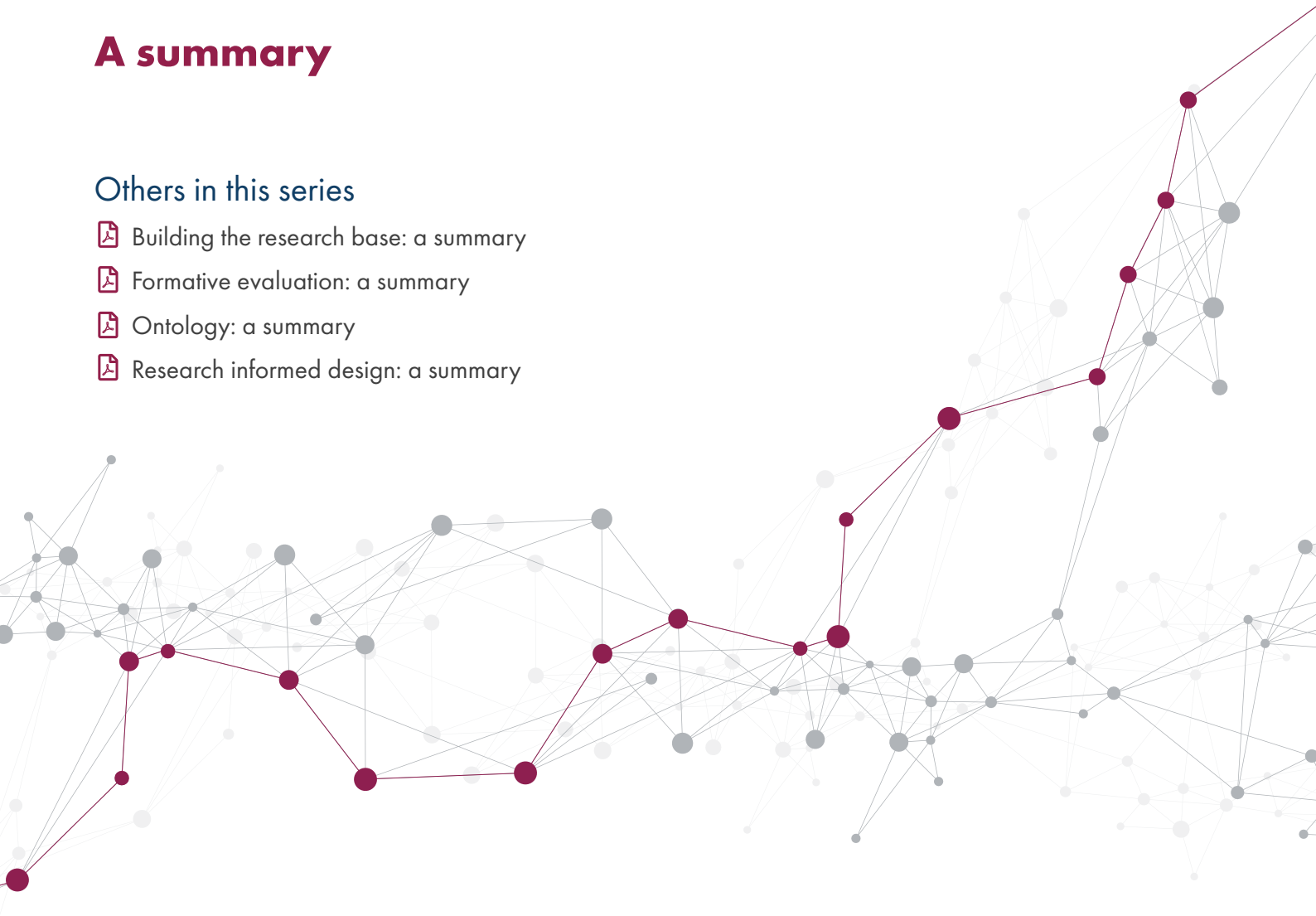
CAMBRIDGE Mathematics

CM Define It

A summary

Others in this series

- Building the research base: a summary
- Formative evaluation: a summary
- Ontology: a summary
- Research informed design: a summary



Author

Dominika Majewska

Representing the work of

Lynn Fortin, Tabitha Gould, Rachael Horsman, Ellen Jameson, Vinay Kathotia, Ray Knight, Lynne McClure, Darren Macey, Dominika Majewska, Nicky Rushton, Lucy Rycroft-Smith and Ben Stevens.

CAMBRIDGE UNIVERSITY PRESS & ASSESSMENT

UNIVERSITY OF CAMBRIDGE Faculty of Mathematics

UNIVERSITY OF CAMBRIDGE Faculty of Education

Introduction and the aims of the project

Investigations of the available literature over the last five years highlighted the lack of a holistic, accessible source of mathematical key terms and definitions, whilst research supports the importance of language in understanding and applying mathematics. Riccomini et al.[1] suggest that several factors are needed in order to successfully communicate using mathematical language; including comprehension skills, numerical proficiency and a solid mathematical vocabulary. In addition to the crucial role that understanding mathematical language plays for learners, mathematical definitions are also very important to the work of teachers and educators. Ball et al. (2008, as cited in Mosvold & Fauskanger[2(p44)]) suggest that a key task for teachers is to select and develop useable definitions. Furthermore, Zazkis and Leikin[3] highlight that the mathematical definition knowledge held by teachers can impact on several aspects of their practice, such as instructional decisions, explanations and guidance offered to learners and the mathematical discussions they lead. Such research demonstrates how crucial knowledge, understanding and access to mathematical definitions can be for educators and its influence on their classroom practice.

The Cambridge Mathematics (CM) team decided to investigate how the mathematics education community perceives existing definitions of key mathematical terminology taken from various international sources. In order to do this, we developed a survey app called *CM Define It*.* The aim of the project was to obtain insight into what mathematics education professionals perceive as successful definitions, with the ultimate, long-term goal being to use the data collected through the *CM Define It* app to inform the glossary layer in the Cambridge Mathematics Framework. Furthermore, we wanted to find out if the constructs in the survey (which represented our expectations for what aspects of definitions might be meaningful to users) could produce meaningful patterns of responses. Data collection for the *CM Define It* project started in October 2019 and ended in December 2020.

* For more details about the development of the survey and the app, including the pilot testing phase, please see Majewska (2019).[4]

CM Define It data collection

Each week, one mathematical key word and up to five definitions were presented, with 62 different words presented in total. We collected users' demographic information, including their occupation, what country they resided in and what groups of learners they worked with from the following:

- novice learners – learners who are developing their early knowledge of core mathematical concepts, e.g. young children;
- intermediate learners – learners who are building on previous knowledge and refining their understanding, e.g. younger teenagers; and
- advanced learners – learners who are comfortable with many core mathematical concepts and are studying or starting to study more advanced or specialised topics, e.g. older teenagers or trainee teachers.[4]

The app presented users with a key word and up to five definitions taken from different international sources. Users were asked to provide each definition with an overall rating on a scale of 1–5. Initially, all users were also asked to provide additional ratings on a scale of 1–5 for the following questions:

- How technically accurate is the definition?
- Doe the definition emphasise key points?
- How accessible is the definition for the chosen audience?
- Is the definition sufficient for the chosen audience?
- Does the definition add to or clarify your own understanding of the word?[4(p7)]

These five criteria used to assess definitions were developed through reading the literature and collaborative thinking regarding the components we would like to know more about, such as language in mathematics education.

However, after three months, the team decided to make additional ratings an optional feature. This decision was made as we recognised that app users may find the process too long. Furthermore, early informal discussions with a mathematics and statistics education expert suggested that the additional ratings made the process of completing ratings very long and could discourage users from participating in the survey app. This further supported our decision to make additional ratings optional. In this newer set up, once users provided an overall rating for the definitions, they were then presented with a unique

mathematical term (such as *camembert*).* They would be given its definition once they provided the additional ratings. This encouraged participation and gave users the option to decide whether or not they wished to provide more detail for each definition.

What did we find out?

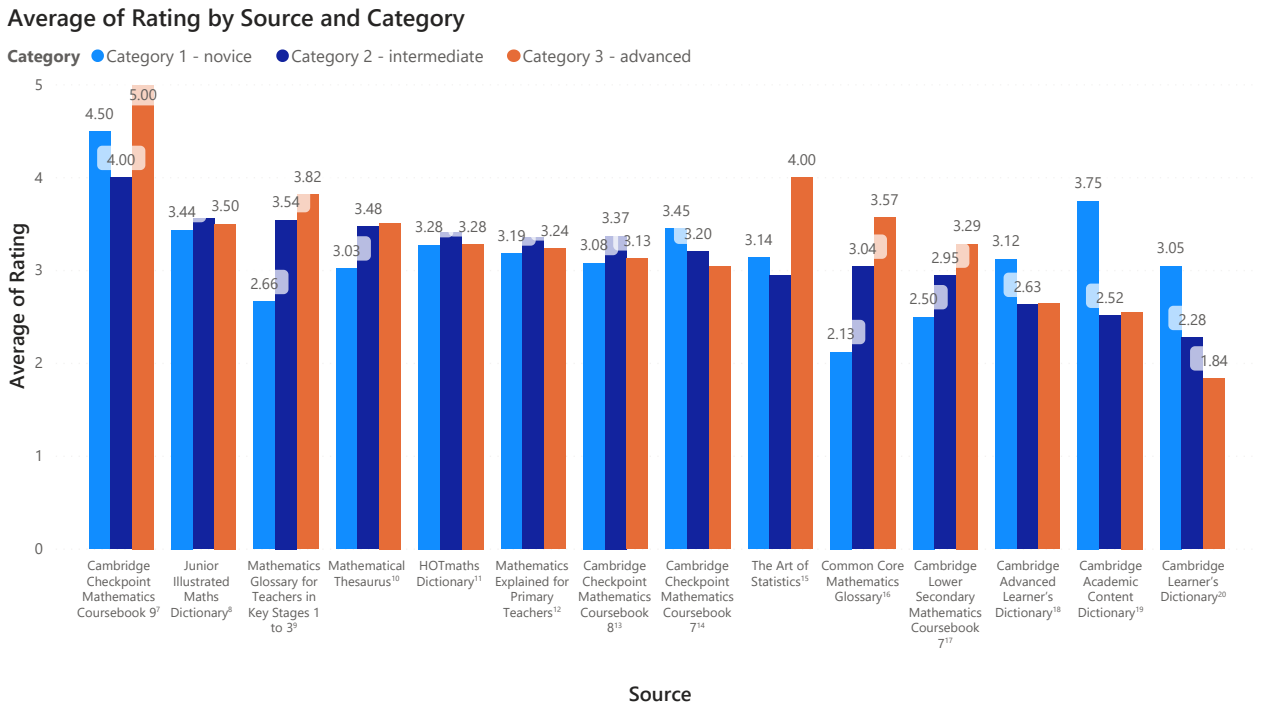
In this short summary we give a very general overview of the survey data. The sample was small and there is no statistical power in the data, therefore no statistical analyses were carried out.[6] The data were exported and presented in PowerBI. We treated the data from this small pilot study as exploratory and will report on some interesting, preliminary observations. Overall, there were 1,217 responses collected between October 2019 and December 2020, but different numbers of responses were collected each week. It is important to highlight that some sources contained more definitions and therefore were used more often, whereas others were used less frequently. It is crucial to keep this in mind when looking at some of the exploratory data, such as average ratings.

When looking at the average ratings given to the sources used, the preliminary data suggest that *Cambridge Checkpoint Mathematics Coursebook 9*[7] was rated the highest by professionals across all three categories of learners. The average rating given by those working with novice learners was 4.5/5, 4/5 by those working with intermediate learners, and 5/5 by those working with advanced learners (see Figure 1). However, it is crucial to consider that whilst 1,217 responses were collected overall, this source was only used once to define the word "prism" and received only 6 responses.

Figure 1 on next page

* Camembert – the French word for pie chart – or, translated more precisely, un diagramme de camembert: a camembert diagram. Camembert is famously round (or wheel-shaped, as we traditionally say for cheese), making it a great metaphor for this type of chart. Other translations refer to pizza or cake instead.[5]

Figure 1. A bar chart presenting the average ratings awarded to each source by category of learners



It may seem surprising that the *Cambridge Checkpoint Mathematics Coursebook 9*[7] received the highest average ratings across all three groups of learners, considering that it is aimed at students of lower secondary school age. It is important, however, to remember that each week, definitions and sources received different numbers of ratings and the average may not reflect this. As this source was used once and collected only 6 responses, the finding and sample can be seen as unrepresentative.

As *Cambridge Checkpoint Mathematics Coursebook 9*[7] had only 6 responses, it is worth looking at the source with the second highest rated average – *Junior Illustrated Maths Dictionary*,[8] which was used to define 32 mathematical words and had 663 responses in total. This source received similar average ratings across all three categories of learners, with those working with novice learners awarding it an average score of 3.44/5, those working with intermediate learners giving it an average score of 3.56/5 and those working with advanced learners giving it 3.5/5.

Upon closer inspection of the five reasons for the ratings, it appears that across all three categories of learners, "How technically accurate is the definition?" was rated slightly higher than the others. How much the definitions added to respondents' own understanding was rated lowest (see Figure 2).

Figure 2. Average ratings awarded to reasons for the scores given to the Junior Illustrated Maths Dictionary[8] *across all three categories of learners*

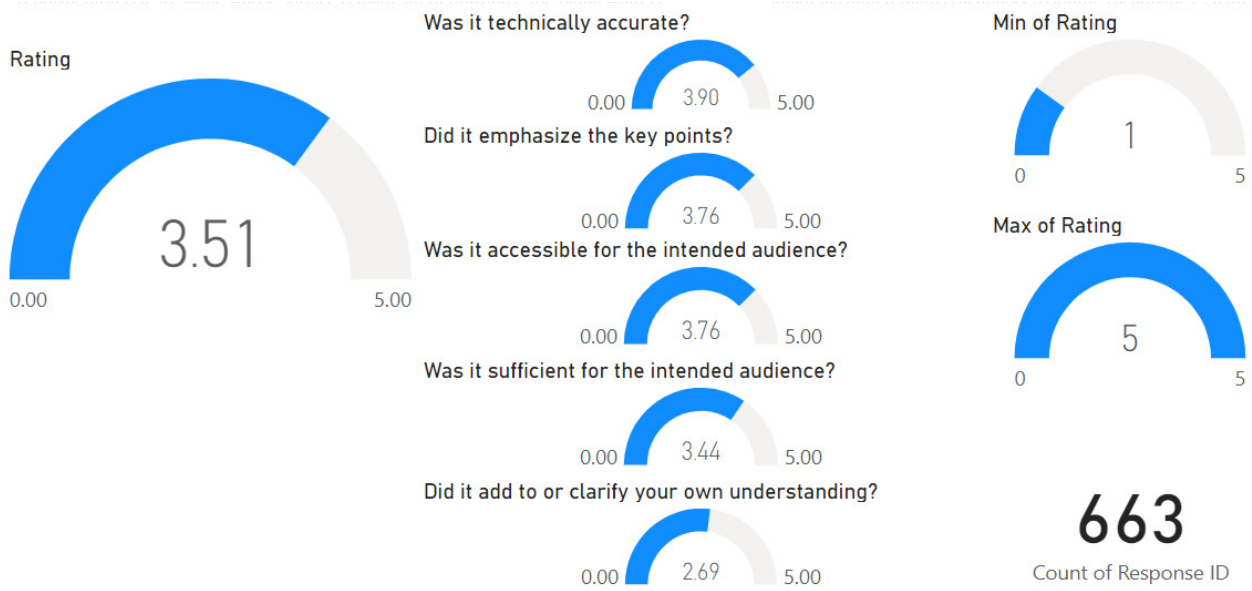


Figure 1 also suggests that *Cambridge Learner's Dictionary* (*CLD*)[20] received the lowest average rating. This may be because this source aims to support learners of British English and is not a mathematics-specific resource. Its definitions may be simpler as the intent is to support those for whom English is not the first language.[20] An interesting observation is that professionals who work with novice learners (category 1) awarded *CLD* an average rating of 3.05 out of 5, which is higher than the average rating given by this group to the definitions from the *Mathematics Glossary for Teachers in Key Stages 1 to 3*[9] (2.66), the *Mathematical Thesaurus*[10] (3.03), the *Common Core Mathematics Glossary*[16] (2.13) and the *Cambridge Checkpoint Mathematics Coursebook 7*[14] (2.5). Could it be that *CLD* breaks down concepts at a level appropriate for young, novice learners, who may be coming across mathematical concepts for the first time? The *CLD* was used to provide definitions for 12 key words and collected 290 responses in total. Upon closer inspection, it appears that those working with novice learners were more likely to give a score of 5 to *CLD* in comparison to those working with intermediate (category 2) and advanced (category 3) learners (see Figures 3, 4 and 5 respectively).

Figure 3. A bar chart presenting ratings given to definitions from the Cambridge Learner's Dictionary[20] *by professionals working with novice (category 1) learners*

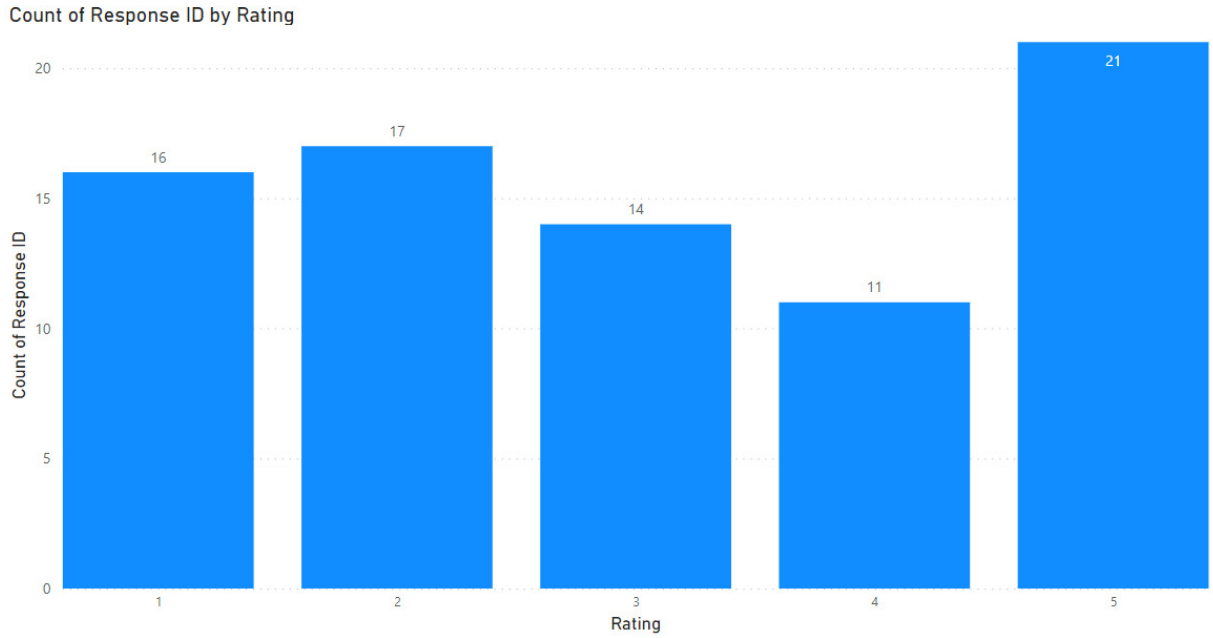


Figure 4. A bar chart presenting ratings given to definitions from the Cambridge Learner's Dictionary[20] *by professionals working with intermediate (category 2) learners*

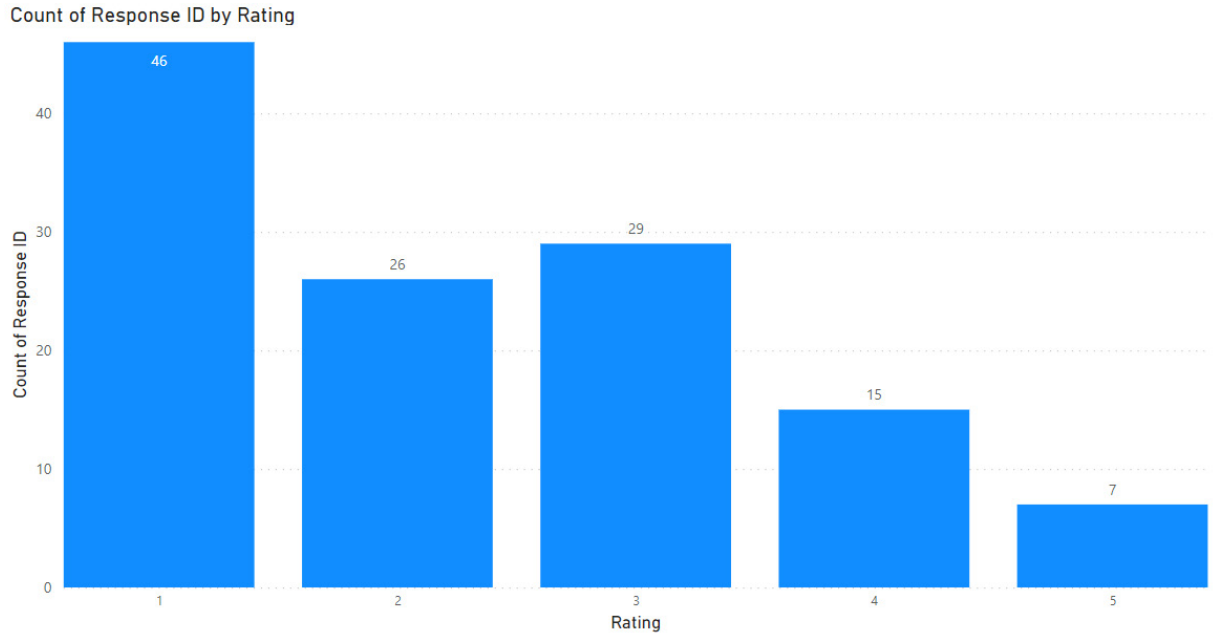
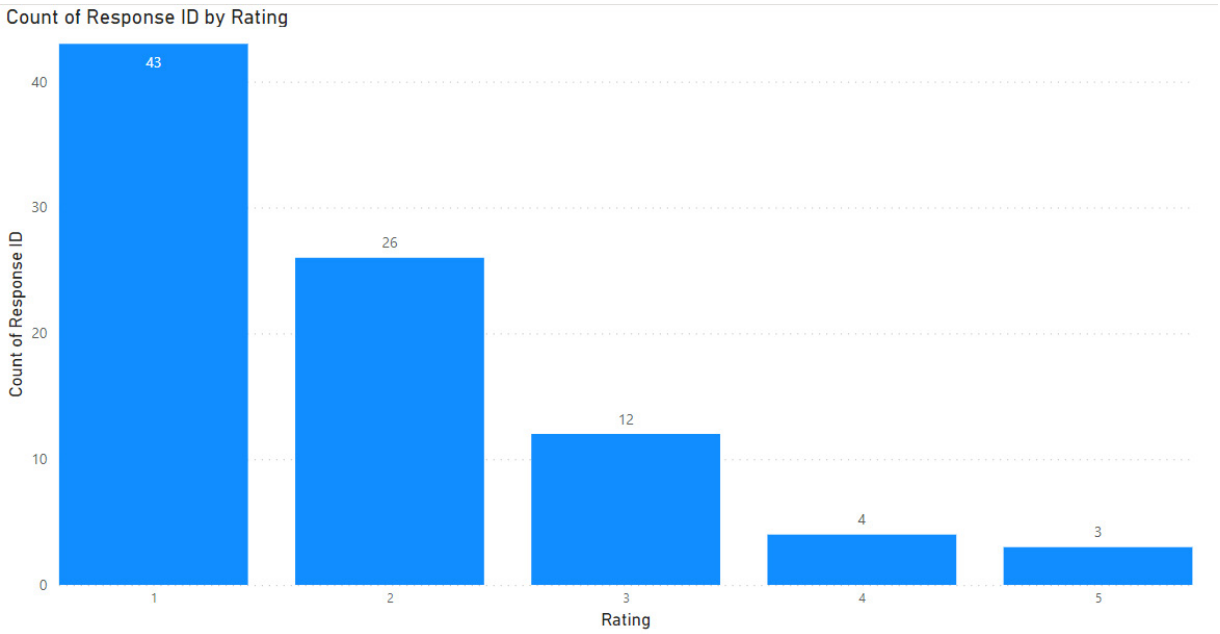


Figure 5. A bar chart presenting ratings given to definitions from the Cambridge Learner's Dictionary[20] *by professionals working with advanced (category 3) learners*



Summary, recommendations and next steps

This short document summarises the background to the *CM Define It* project, its aims and goals, and gives insight into the kind of information that was collected and how we've explored the collected data. In this summary, we decided to focus on exploring the average top-rated sources and the lowest-rated source. The collected information suggests that:

- Although the *Cambridge Checkpoint Mathematics Coursebook 9*[7] received the highest average ratings as a source, it was used to define only 1 word and received only 6 responses, making it unrepresentative.
- The source with the second highest average rating (*Junior Illustrated Mathematics Dictionary*),[8] which was used to define 32 words and received 663 responses in total, showed similar average ratings across the three categories of learners. Upon closer inspection, it appears that across the three groups of learners, the technical accuracy of the definitions from this source received a higher rating (3.9/5) than other criteria on which sources were judged. It may be worth investigating these exploratory findings in more detail to find out what specific aspects made the definitions accurate and whether users perceived some elements as particularly useful in defining mathematical key words.

- Although the *CLD*[20] was awarded the lowest average rating, it may be useful to investigate why those working with category 1 learners gave it an average rating of 3.05/5, which is higher than the average rating of other sources, such as the *Mathematics Glossary for Teachers in Key Stages 1 to 3*.[9] More in-depth, qualitative research could be useful in answering some of these questions. The findings of such research could shine light on what is perceived as more useful for younger learners in comparison to older learners in the context of mathematical definitions.
- An important aspect is how useful this pilot project was in testing the construct of perception of mathematical definitions. It enabled us to see some differences between groups of users in how they responded to some sources according to the dimensions we suggested. A study with greater statistical power would be required to draw further conclusions about definitions of mathematical key terms.

As research highlights the importance of language in learning and teaching mathematics, researchers should investigate further the specific aspects of mathematical definitions which would be beneficial to students who are at different stages of learning. Ball[21(p6)] argues that "a definition of a mathematical object is useless, no matter how mathematically refined or elegant, if it includes terms that are beyond the prospective user's knowledge." It would be beneficial for those developing mathematical definitions and for those using such resources in practice, to better understand the needs of learners at different stages of their journey through mathematics education. Such research would have valuable implications for many, including teachers, resource developers and designers, researchers and students. The CM team will investigate the information collected through the *CM Define It* survey app more closely to see if and how the findings from the *CM Define It* survey app can inform the glossary embedded in the Cambridge Mathematics Framework.

Recommendations:

- An investigation into the general glossary design is needed to deliver more targeted and appropriate definitions for different learners.
- A more specific, focused inquiry into the specifics of definitions and types of information that can benefit students at different stages of learning is needed.
- The Cambridge Mathematics Framework could be used by curriculum/glossary professionals to check and refine appropriateness of definitions for learners.

A key conclusion from this initial study is that due to the volume of information, it may not be feasible to create one single glossary for all who are working with mathematical definitions. Findings regarding the needs of different learners should be used to inform future glossaries of mathematical definitions to make them as suitable and appropriate for learners as possible. With respect to the Cambridge Mathematics

Framework, the initial findings suggest that ideally multiple sets of definitions would be available in the glossary layer, appropriate for specific uses. Despite whether or not this will be possible in the future, we are enhancing our understanding of differences in the language needs of educators who work with diverse groups of learners.

References

1. Riccomini, P. J., Smith, G. W., Hughes, E. M., & Fries, K. M. (2015). The language of mathematics: The importance of teaching and learning mathematical vocabulary. *Reading & Writing Quarterly, 31*(3), 235–252. https://www.tandfonline.com/doi/abs/10.1080/10573569.2015.1030995?journalCode=urwl20
2. Mosvold, R., & Fauskanger, J. (2013). Teachers' beliefs about mathematical knowledge for teaching definitions. *International Electronic Journal of Mathematics Education, 8*(2-3), 43–61. https://www.iejme.com/download/teachers-beliefs-about-mathematical-knowledge-for-teaching-definitions.pdf
3. Zazkis, R., & Leikin, R. (2008). Exemplifying definitions: A case of a square. *Educational Studies in Mathematics,* 69(2), 131–148. https://link.springer.com/article/10.1007%2Fs10649-008-9131-7
4. Majewska, D. (2019). *Glossary app: The development and pilot phase of CM Define It*. Cambridge Mathematics. https://www.cambridgemaths.org/research/framework-documentation/view/glossary-app-the-development-and-pilot-phase-of-cm-define-it/
5. Rycroft-Smith, L. (2020). *Maths words for glossary app*. Unpublished document.
6. van Teijlingen, E., & Hundley, V. (2001). The importance of pilot studies. *Social Research Update*, 35. http://sru.soc.surrey.ac.uk/SRU35.html
7. Byrd, G., Byrd, L., & Pearce, C. (2013). *Cambridge Checkpoint mathematics coursebook* 9. Cambridge University Press. https://www.cambridge.org/gb/education/subject/mathematics/cambridge-checkpoint-mathematics/cambridge-checkpoint-mathematics-coursebook-9-digital-coursebook-9-1-year?isbn=9781107668010&format=PB
8. Large, T., & Rogers, K. (2010). *Junior illustrated maths dictionary* (Usborne Dictionaries). Usborne.
9. National Centre for Excellence in the Teaching of Mathematics. (2014). *Mathematics glossary for teachers in Key Stages 1 to 3*. https://www.ncetm.org.uk/media/hpihrj3s/national-curriculum-glossary.pdf
10. University of Cambridge & Partners. (2001–2004). *Mathematical thesaurus*. EU Socrates Minerva project. Millennium Mathematics Project.
11. HOTmaths. (n.d.). *Dictionary*. https://www.hotmaths.com.au/usersCommon/loadGlossary.action
12. Haylock, D. (2006). *Mathematics explained for primary teachers* (3rd ed.). Sage.
13. Byrd, G., Byrd, L., & Pearce, C. (2012). *Cambridge Checkpoint mathematics coursebook 8*. Cambridge University Press. https://www.cambridge.org/gb/education/subject/mathematics/cambridge-checkpoint-mathematics/cambridge-checkpoint-mathematics-coursebook-8-digital-coursebook-8-1-year?isbn=9781107697874&format=PB

14. Byrd, G., Byrd, L., & Pearce, C. (2012). *Cambridge Checkpoint mathematics coursebook 7*. Cambridge University Press. https://www.cambridge.org/gb/education/subject/mathematics/cambridge-checkpoint-mathematics/cambridge-checkpoint-mathematics-coursebook-7-digital-coursebook-7-1-year?isbn=9781107641112&format=PB
15. Spiegelhalter, D. (2019). *The art of statistics: Learning from data*. Pelican.
16. Common Core State Standards Initiative. (n.d.). *Mathematics glossary*. http://www.corestandards.org/Math/Content/mathematics-glossary/
17. Byrd, G., Byrd, L., & Pearce, C. (2021). *Cambridge lower secondary mathematics coursebook 7* [First draft]. Cambridge University Press.
18. Cambridge University Press. (2013). *Cambridge advanced learner's dictionary*. https://www.cambridge.org/gb/cambridgeenglish/catalog/dictionaries/cambridge-advanced-learners-dictionary-4th-edition
19. Cambridge University Press. (2008). *Cambridge academic content dictionary*. https://www.cambridge.org/gb/cambridgeenglish/catalog/dictionaries/cambridge-academic-content-dictionary
20. Cambridge University Press. (n.d.). *Learner's dictionary*. https://dictionary.cambridge.org/dictionary/learner-english/
21. Ball, D. L. (2003). *What mathematical knowledge is needed for teaching mathematics?* [Paper presentation]. Secretary's Summit on Mathematics, U.S. Department of Education, Washington, D.C., USA. http://jwilson.coe.uga.edu/Situations/Framework%20Folder/Framework.Jan08/articles/Ball2003Math%20Summit.pdf