CAMBRIDGE Mathematics

Case study report

Designing a framework for computational thinking with Arm

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Introduction

This micro-report describes a framework design case study featuring the use of the Cambridge Mathematics (CM) Framework ontology design tools. We present the background for our series of case study micro-reports separately[1]. This case allowed us to explore the extent to which elements of the ontology for the CM Framework could be used to develop a framework for computational thinking (CT). It also gave us the opportunity to observe how the ontology design tools we developed for ourselves when designing the CM Framework might be used by someone else with a different set of goals and constraints.

Case context and goals: exploring new domain representations with the tools and structure of the CM Framework

Arm is a multinational company which designs widely-used semiconductors. We collaborated with a researcher from the Arm Schools Program, part of the Arm Education Group. The program aims to support professional development in STEM education through research, community-building and resource development. The Arm researcher's goal for this collaboration was to identify and represent elements of CT in a framework which could be used to structure support for CT in Arm's STEM education resources.

The Arm researcher used the process of developing a CT framework to explore two conceptions of CT which have distinct implications for representing it as a framework, relating it to STEAM domains and integrating it into a curriculum. A *domain-general* perspective on CT involves abstracting ways of thinking about, and doing, computation to describe what these activities have in common across contexts. From this perspective CT is an approach to solving complex problems and may be used in diverse areas (Lai, 2019c). A *domain-specific* perspective on CT focuses on CT as embedded in specific domain contexts; i.e. within particular topics or activities within computer science, mathematics or other STEAM domains (Lai, 2019a, 2019b). Both perspectives on CT may be important to consider when representing it across subjects and curricula.

There are clear parallels with notions of 'mathematical thinking' in this regard: in curriculum frameworks, mathematical thinking may be represented in a general way, an embedded way, or both. Sometimes when it is described more generally, actionable links to specific mathematical ideas

[1] See Background for the case study report series (Jameson, 2019a)

are not always provided. When it is intentionally embedded throughout a curriculum, the big picture of the development of mathematical thinking through engagement with mathematical ideas year by year is unlikely to be automatically visible to teachers or students. Similarly, curriculum frameworks may be too high-level to represent embedded CT effectively, given that CT is relatively new and some stakeholders may be unsure how to embed it effectively into their curriculum frameworks (e.g. deciding whether to embed it as a skill applied across subjects, or tied to particular subjects, or both). If curriculum development at that level of detail is left up to smaller regions within the overall jurisdiction a curriculum framework applies to, the links between general descriptions and specific instances of mathematical thinking may be lost in translation. Teacher experience and specialisation may or may not be able to fill in the gaps.

The Cambridge Mathematics team had previously explored aspects of representing mathematical thinking as part of a Delphi study conducted in 2018 with an international panel of senior mathematics curriculum researchers and developers. This left us with an understanding of areas of consensus and disagreement among panel participants around priorities for representing mathematical thinking in the CM Framework. It also sparked discussions about possible design solutions; e.g. highlighting mathematical thinking in professional development built on top of (and mapped to) the CM Framework, and/or linking it to examples of how students act on mathematical ideas which are already embedded throughout the CM Framework (see *student actions* under Cambridge Mathematics Framework structure and content).

Finally, the use of technology in mathematics education renders computational thinking relevant in mathematics education in its own right. The PISA 2021 Mathematics Framework reflects this by highlighting the emerging role of computational tools and computational thinking in mathematical literacy (OECD, 2018). Therefore, one of our design goals for the Cambridge Mathematics Framework is to provide flexibility for teachers and educational designers to change the order or pedagogical approach they use to engage students with particular mathematical ideas if they are using technological tools which support this. An exploration of computational thinking conducted using our CM Framework design tools, and with our ontology in mind, could prepare us when developing pedagogical support for uses of technology in the future.

The CM team therefore viewed this collaboration through the lens of the following design questions:

1. How might CT be represented as a framework without embedding it completely in a domain-specific framework (e.g. computer science) analogous to the CM Framework?
2. Does the process of designing a framework to represent CT suggest anything about a process we might explore further for representing mathematical thinking in the CM Framework?

3. Are elements of the ontology (structure and meaning of content in the CM Framework) which we have developed for representing mathematical ideas useful for representing CT?
4. Can the tools we have built to design our ontology be used to develop another?

With respect to question 1, the Arm researcher sought to identify and develop the structure, core themes and key components of CT within her framework. This included the investigation of possible overlaps in structure and content between mathematics and CT.

More generally, the CM Framework can be characterised as a type of professional *knowledge map* (a structured, networked way of communicating professional knowledge), and we wanted to observe whether the tools we used to design it could also be used to create other knowledge maps successfully. Table 1 shows the potential affordances of knowledge maps which guided our observations in this case.

Table 1: Potential affordances of knowledge maps

	Combined user groups	This case
Help designers to communicate ideas about knowledge to others; make tacit ideas explicit and present ideas in a form that users can relate to (Eppler, 2004; Vail, 1999)	✓	✓
Help users to "remember, comprehend, and relate knowledge domains through insightful visualization and aggregation of information" (Eppler, 2004, p. 200)	✓	✓
"[M]ake information actionable in new contexts, connect it with previous experiences" (Eppler, 2004, p. 189) – that is, professional learning and transfer	✓	
Help users to evaluate what knowledge is available for decision-making, and from what sources (Eppler, 2004)	✓	
Help users to see concepts within a bigger picture and to switch between multiple perspectives (Eppler, 2004)	✓	✓
Help users to evaluate and compare sets within knowledge domains – examining what knowledge is available, from what sources, and with what justification (Eppler, 2004)	✓	
Provide a "common framework" when searching for or contributing "relevant knowledge" (Eppler, 2004, p. 190), which itself supports professional learning	✓	✓
Contribute to the field by providing a big-picture perspective and a research base with respect to ideas that people in different roles may hold in common	✓	✓
Relate the big-picture perspective to different levels of underlying detail (Eppler, 2004)	✓	✓
Support professional learning in practical contexts: "just-in-time" (Vail, 1999, p. 23)	✓	

Role of this study in our evaluation plan

The Arm CT framework development study has helped us to do some initial exploration of several open questions around the applicability of our work outside the mathematics education domain. It has given us a chance to explore

- whether elements of the CM Framework might be used directly to represent related but different CT ideas in STEAM,
- whether someone with different design goals and constraints, focusing on knowledge from a different domain with potentially a different structure, can use our design tools to develop a framework in the form of a knowledge map, and
- what ideas a CT framework might suggest for the structure or visualisation of content as we plan our representation of mathematical thinking.

Methods and materials

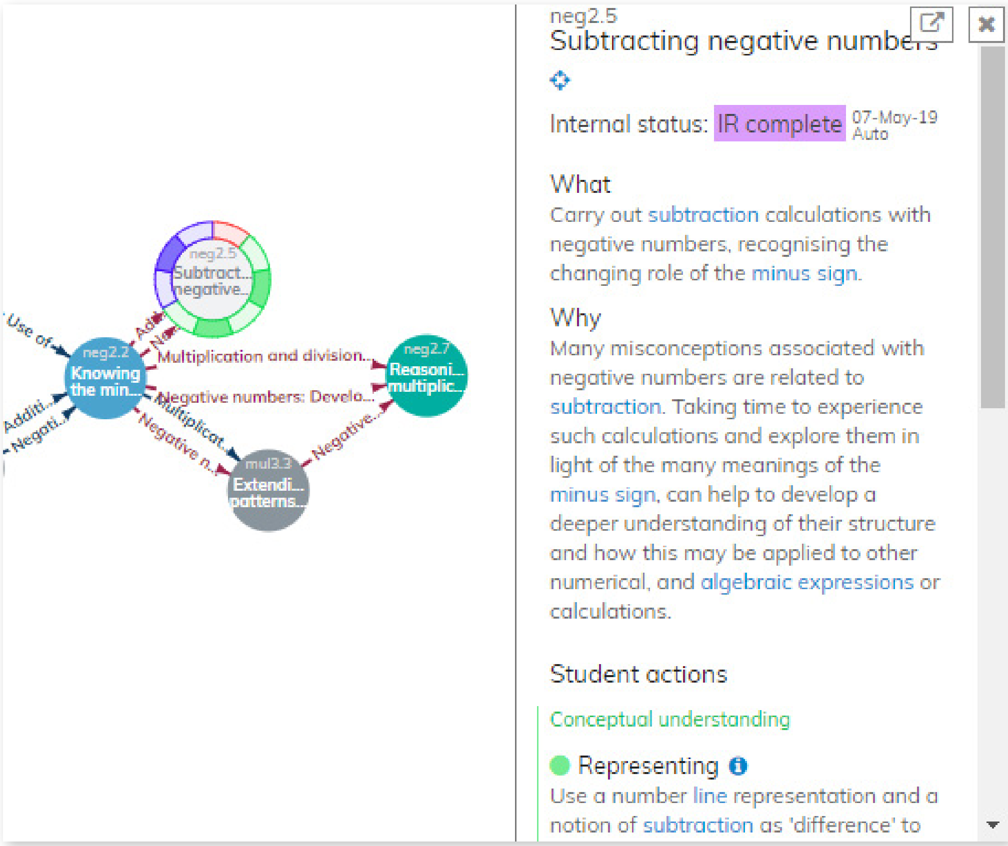
Cambridge Mathematics Framework structure and content[2]

The CM Framework team used the network of mathematical experiences in the CM Framework which they developed from their interpretation of empirical research and the knowledge of expert collaborators who are familiar with both research and practical teaching contexts in multiple jurisdictions. In the CM Framework, mathematical ideas and key relationships between them are represented in a network of *waypoints*, which we define as "places where learners acquire knowledge, familiarity or expertise" (Jameson et al., 2019, p. 4). Waypoints have titles, descriptions and *student actions*, which are examples of the kinds of things students might do to help them build an understanding of the content at a waypoint. An example of a waypoint and its content is shown in Figure 1. We call the relationships between waypoints *themes*, and in the CM Framework they represent either the development of some part of an idea from one waypoint to the next, or the use of one idea contributing to the understanding of another[3].

[2] Parts of this subsection are reproduced from Case study micro-report: Mapping tasks to the Cambridge Mathematics Framework with Vretta (Jameson, 2019b)
[3] Waypoints, themes, and types of student actions are described in Ontology: Structure and meaning in the Cambridge Mathematics Framework (Jameson et al., 2019)

Some waypoints play special structural roles. Exploratory waypoints "often come at the beginning of a theme…and indicate a place where ideas can be played with in a less formal or more playful way, as part of building mathematical intuition." Landmark waypoints are places where "ideas are brought together such that the whole experience may seem greater than the sum of its parts" (Jameson et al., 2019, p.6)

Figure 1: Example waypoint with content ("what," "why" and "student actions")



Research Summaries are documents in which a subset of this network of waypoints and relationships is embedded; they "tell the story" of the content and structure of a set of waypoints and themes. Each Research Summary includes three potential elements of interest for this case:

1. a review of the literature informing the content,
2. the embedded CM Framework content and
3. a description of how our interpretation of the sources in our research base has led to the structure shown in that Research Summary.

We determined that Research Summaries would be the most accessible way, at our current stage of interface development, to present the CM Framework content for external use.

The CM Framework is designed, authored and viewed through the online CM Framework platform, CMF Nexus (Stevens et al., 2019). This platform and the initial structures of waypoints and themes in the CM Framework were used as a starting point for the development of the CT framework.

Team backgrounds

The ARM researcher whose work is reported here was a PhD student in the Faculty of Education at the University of Cambridge. She was able to draw on the literature review already underway for her thesis on computational thinking and was familiar with a range of issues around CT in STEM education. The backgrounds of the CM Framework design team as a whole involved multiple professional roles in mathematics education and research. These included mathematics teaching and teacher professional development, resource development and educational design in mathematics and STEM, qualifications development in mathematics, and educational research in STEM learning.

Pilot case protocol

The Arm researcher began by presenting her initial research to the CM team and discussing possible directions in which work might proceed. While using CMF Nexus, she worked with face-to-face access to the CM team and discussed writing, methodology and technical questions with us as needed. She documented and explained the CT framework she developed in notes as she worked, which she compiled into a report and final presentation at the end of the project. CM team members made note of particular questions she asked or thinking she described while using the design tools in CMF Nexus and referencing CM Framework content.

Time frame

The entire project took approximately three months, during which time the Arm researcher worked an average of 20 hours/week alongside the CM team.

Results

This paper is focused on the implications of this case for the CM Framework and design tools in CMF Nexus rather than the Computational Thinking (CT) framework itself. The Arm researcher presented the details and justification for the design of the resulting CT framework separately at a CIDREE STEAM expert meeting (McClure & Lai, 2019) and in an internal report for Arm (Lai, 2019a). These will be summarised but not reported here in full.

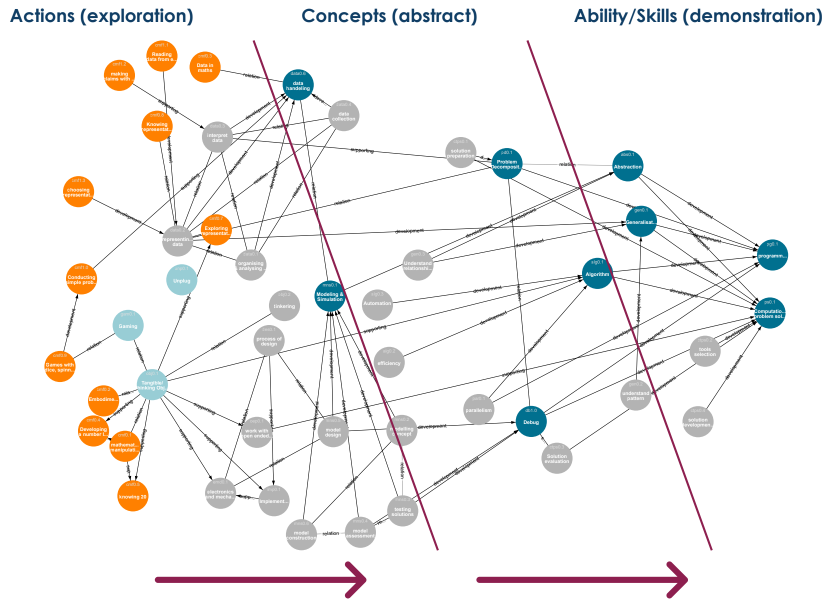
The framework for CT developed in this case is an initial exploration of the representation of CT for educational design and support. It has not been validated or published. In its current form, it will contribute to ongoing work in CT within the Arm Schools Program and may form the basis for further framework development.

The Computational Thinking (CT) Framework

The CT Framework, developed with the graph database knowledge mapping tools in CMF Nexus, is naturally similar to the structure of the CM Framework in that it possesses the basic structural elements of a graph: it has *nodes* (points in the graph which represent some piece of knowledge) and *edges* (connections between points indicating some sort of relationship) (see Figure 2). The nodes, or CT waypoints, are distinct from CM Framework waypoints because they represent areas that can be linked to and extend from the CT Framework to the CM Framework. The edges, as in the CM Framework, can be either directed, indicating that one idea contributes to another in some way, or undirected, indicating that a relationship has been identified by the designer but the nature of it is still ambiguous (Lai, 2019a).

Figure 2 on next page

Figure 2: The size and structure of the CT Framework (reproduced from Lai, 2019a, p. 4). Orange nodes are CM Framework waypoints, light blue are exploratory CT waypoints, dark blue are landmark CT waypoints, and grey are standard CT waypoints



The waypoint types from the CM Framework were adopted but they were used in quite a different way to represent elements of CT. A general sense of having exploratory waypoints to represent informal exposure to ideas and of having landmark waypoints to represent places where several ideas are brought together was retained. However, as CT waypoints tend to build up from the left to right side of the framework, the nature of the waypoints seemed to evolve in different stages: those on the left (more foundational in terms of dependencies) tended to represent general types of actions and explorations (e.g. "gaming"); those in the middle tended to represent concepts that are likely more abstract (e.g. "parallelism"); and those towards the right (which depended most on waypoints which came before) represented complex abilities and skills (e.g. "generalisation") (Lai, 2019a). The Arm researcher discovered a parallel between this representation of CT and Bloom's Taxonomy, a hierarchical representation of educational learning goals with higher-order goals building on lower-order ones (Webb, 2014). She described this as unexpected because she had not explicitly intended to portray CT in terms of developmental progression, given that there is not clear support in the literature for doing so (Lai, 2019a).

The designation of different types of possible edges and their defined meanings in the network emerged as an important design feature for placing the focus on relationships between elements of CT in the CT

Framework. A new directed edge type called *Supporting* was invented to indicate CT waypoints "that strengthen or assist other waypoints but do not necessarily emerge as or develop into them." (Lai, 2019a, p. 3). The essence of "Development of" edges remained the same. The meaning of "Use of" edges in the CM Framework was tweaked into *Relation* edges, allowing the CT framework to represent relationships without making further claims about the nature of the relationships (e.g. development, support, use of one idea when engaging with another). The Arm researcher noted that she tended to use these "Relation" edges for tight clusters which develop into a CT landmark waypoint. Unlike the CM Framework, edges were not further classified according to theme, so themes were not available as a means of grouping nodes or distinguishing between paths (Lai, 2019a).

"Development of" edge types and landmark waypoints helped to develop the CT Framework's hierarchical structure. The Arm researcher considered landmark waypoints to be higher-order because they could be taken to represent what emerged from the set of CT waypoints feeding into them. The left-to-right shift from actions to concepts to identification of demonstrable skills and abilities functions as another kind of hierarchy, with waypoints describing abilities growing out of the network of concepts and actions which came before (Lai, 2019a). These two hierarchical elements of the CT Framework's structure are also present in the CM Framework.

The Arm researcher found it meaningful that certain CT concepts such as data handling (e.g. "Conducting simple probability experiments") and tangible objects (e.g. "Mathematics manipulatives") can be associated with and linked to the CM Framework. As shown in Fig. 2, she placed them entirely in the exploratory (action) zone of the CT Framework, which she explained was due to the more concrete nature of this zone. She reported that these were included as a proof of concept and that many more such ideas from mathematics and other domains could be mapped on to the CT Framework (Lai, 2019a).

Process

The Arm researcher converged on the final structure of the CT Framework by working both from the top down, exploring overarching themes, and the bottom up, identifying and structuring individual components of CT. Both approaches were informed by literature review (Lai, 2019a). As she worked, she was able to use the design tools in CMF Nexus to create and try out new features for the CT Framework which were ultimately adopted into its final structure.

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Discussion

Structure and process

The CT Framework ontology developed by the Arm researcher differed from that of the CM Framework in structure and application. Structurally, features of the CM Framework were variously added, adapted and removed, producing a different ontology to serve a different purpose. The CT Framework ontology was also applied to the content of the CT Framework in a different way. The CM Framework ontology was designed to help multiple writers to apply consistent meaning and scope in the way ideas are represented within waypoints; this not only helps them to coordinate but also helps to increase clarity of meaning for external audiences across a large number of topic areas in mathematics. The CT Framework, in contrast, somewhat shifts the meaning of its waypoints from left to right, reflecting the range of different types of CT elements described in the literature.

These differences can be interpreted in light of the different goals, starting materials and constraints of the two projects. The CM Framework involves the coordination of multiple writers and a large number of content-specific waypoints and relies on additional representations to show the development of ideas, whereas the CT Framework is designed to provide a higher-level overview of the elements of CT. The CM Framework is completely domain-specific, while the CT Framework includes domain-specificity mainly as a proof of concept and is therefore structured in a more domain-general way. The CT Framework is designed to help structure professional development and the design of classroom resources according to a big-picture sense of CT, while the CM Framework also needs to support coherence in detailed curriculum development across the mathematics education domain. Finally, mathematics is a domain which has developed over a long period of time whereas CT is not itself a domain and is in its infancy by comparison; each approach to framework development can be appropriate according to its purpose.

The parallels the Arm researcher saw with Bloom's Taxonomy suggest that her approach to a CT framework tended towards using progressive relationships between CT elements to illustrate aspects of the overall development of CT. Since the tools in CMF Nexus afforded the use of either directed or undirected edges, and the Arm researcher began the project with an understanding of progression in CT as an open question in the literature, this design choice is likely to have been influenced more by the CT literature than by the design tools used to develop the CT Framework. On the other hand, the researcher also noted an alignment and compatibility between the structure of the CT Framework with constructionist and constructivist learning theories, particularly with the idea of learning by exploring, constructing and building (Papert & Harel, 1991). This is expressed by the transition from exploratory waypoints to standard and landmark waypoints in the CT Framework.

A similarity emerged in requirements for the development process. We had previously worked with an Arm researcher who was an engineer but did not have a background in computer science education or in educational research. While this background gave her an expanded perspective on some of the end goals for a CT Framework, she found that there was not enough information in the CT education literature alone to structure the development of a framework for use by educational designers and teachers. After a preliminary trial it was decided that experience with CT/CS in education contexts was important for interpreting the research on CT to create a useful framework for education. The Arm researcher featured in this case had this experience in the form of classroom implementations for educational research. In the case of the CM Framework, members of the CM team interpret mathematics education research through the lens of their classroom teaching experience. Success in approaching framework development in both cases suggests that a wide variety of experiences might contribute to designing this type of framework.

User actions

This case differs from other case studies reported in this series in that it does not describe a scenario involving what we see as a key use of the CM Framework upon launch. In consequence, we have not focused our discussion on user actions identified in this case. Nevertheless, one action stood out as being unique to the ontology development process described here: "adopt and adapt." CMF Nexus provides a flexible feature-labelling system which lets the CM Framework designers create a set of names they wish to apply to different features. This labelling has been altered earlier in our design phase as the need for modifications to the ontology became apparent. These labelling features allowed the Arm researcher to start with nodes and edges which had one meaning in the CM Framework ontology and then to make changes, additions and removals according to what was needed.

Overlaps between CT and mathematics

As we explore cross-disciplinary connections in the future, we will be able to focus on some of the possible areas of overlap identified in this case. After the work described in this paper had taken place, it was presented at a CIDREE STEAM expert meeting. The meeting gathered curriculum experts and representatives of different European countries to discuss their vision of how CT could and should be embedded into the mathematics curriculum. (McClure & Lai, 2019). This group identified "decomposition, pattern recognition, abstraction, and algorithms" as elements of computational thinking which could particularly be integrated into mathematics curricula, whether through problem-solving or programming, with or without the use of technology (Lai, 2019c). It was discussed that these elements may be applicable and relevant to graph theory, statistics, set theory and logic (McClure & Lai, 2019). In this sense, CT can be viewed as a tool to solve problems in mathematics. Another discussion in the meeting focused on the similarities and differences between mathematical thinking and computational thinking: whether the two terms are used to explain similar processes. There are diverse opinions on this, possibly due to the different backgrounds of the participants. Nonetheless, it was agreed that mathematical thinking and computational thinking share several core common processes.

As we work towards a representation of mathematical thinking in the CM Framework, this case has helped us to take stock of some of our options – we could integrate it into the waypoints layer, or create new features for expressing it across waypoints. Because we use our ontology to divide up different kinds of content so that meanings can be expressed consistently across a large number of nodes, we are exploring options like representing mathematical thinking in professional development so that it can be exemplified with mathematical ideas which are appropriate to the topic of various professional development activities. We are also exploring how mathematical thinking might be expressed in a layer of tasks which themselves would be linked to waypoints.

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