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CAMBRIDGE √Mathematics

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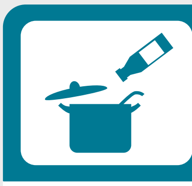
TALKING POINT:

WHAT DOES RESEARCH SUGGEST ABOUT THE DEVELOPMENT OF PROPORTIONAL REASONING IN MATHEMATICS LEARNING?

IN SUMMARY

- Proportional reasoning involves an understanding of proportionality – change and invariance in relationships – and should be integrated and connected across mathematical strands
- Allowing students a wide range of proportional reasoning experiences over a number of years, beginning when they are young alongside development of their rational number sense, is recommended
- Proportional reasoning is more than just finding missing values; it is a lens for problem-solving that lays important foundations for algebraic thinking
- Premature memorisation of rules is likely to inhibit development of proportional reasoning
- Students should have opportunities to sketch, describe and represent proportion problems and relationships between quantities in informal, invented ways before moving towards symbols and algebra
- Students should be encouraged to explore different areas of mathematics (including slope, scale, probabilities, vectors and rates) through a lens of proportional reasoning, finding the connections and similarities

Two possible visual representations of proportional reasoning



Ant is making spicy soup for 11 people. He uses 25ml of tabasco sauce.

Bea is making the same soup for 33 people.

How much tabasco sauce should she use?

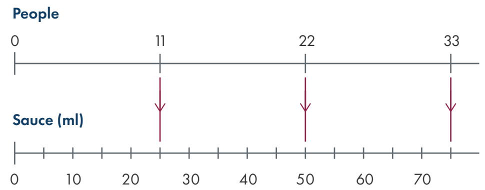
Exploring multiplicative relations *within* a measure space (scalar relation)

Exploring multiplicative relations *across* a measure space (function relation)

RATIO TABLE

Number of people	11	22	33
Sauce (ml)	25	50	75

DOUBLE NUMBER LINE



Adapted from Küchemann et al. (2014)[15]

1

Proportional reasoning – categorising, understanding and manipulating multiplicative relationships – is core for all mathematics learning.[3] Proportional reasoning is the act of using and understanding *proportionality*, in other words "being able to make comparisons between entities in multiplicative terms."[4] Proportionality can be static (pairs of: values, fractions, or ratios) or dynamic (linear functions, rates of change)[5] and exploring both flexibly is important. It involves recognising the difference between absolute and relative change and understanding that the relationship between quantities in a ratio is invariant,[6] as well as considering the appropriateness of assumptions when characterising such a relationship.[7] It is a key connection between topics across contexts in number, geometry, probability and statistics and therefore should be integrated into learning in all of them.[8] It does not develop easily or quickly, requiring a wide range of experiences over a number of years to achieve proficiency.[7]

IMPLICATIONS: Proportional reasoning is key to mathematics learning, involving an understanding of change and invariance

Proportional reasoning should be integrated and connected across mathematical strands such as number, geometry, probability and statistics

Students should have a wide range of proportional reasoning experiences over a number of years in order to develop proficiency in its use

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Proportional reasoning develops alongside rational number sense[9] and is rooted in early (pre-school) experiences[10] such as comparing, measuring, building up, sharing, growing and shrinking. It also relies on an understanding of dividing (splitting into equal parts) and unitising (measuring several equal parts as one unit). Exploring problems through a lens of proportional reasoning allows pupils to form a framework for them to think about and justify relationships, which may help prevent numerical computations "materializing from nowhere."[7] Careful choice of values used in problems may help provoke a need for multiplicative reasoning. Teachers have a tendency to over-interpret students' responses and judge that they have understood proportionality on the basis of a correct answer.[11] Premature use of "shortcut" rules may form a barrier to the development of proportional reasoning as students may apply them without thinking[4] — because teaching of proportional reasoning is often limited to memorising procedures for solving missing value problems.[7]

IMPLICATIONS: Young children can begin to explore early ideas of proportional reasoning alongside the development of rational number sense, building on dividing and unitising

Successful proportional reasoning is not necessarily demonstrated by a single correct answer

Memorisation of rules may encourage students to apply rules without thinking and therefore inhibits development of proportional reasoning

3

Proportional reasoning, in the sense of "an early emphasis on developing children's ability to conceive of, reason about, and manipulate complex ideas and relationships," is likely to enable students to better understand algebra.[7] Proportional reasoning and algebraic reasoning are intrinsically linked, particularly as the model for a linear function is built on recognising a proportional relationship.[12] Students should have opportunities to explore within a measure (scalar relation) and across measures (function relation); see infographic.[1]

IMPLICATIONS: Proportional reasoning is a key part of the basis for algebraic thinking and the two should be explicitly linked for students

4

When considering proportional problems, students should have opportunities to explore and represent the problem situation and the quantities involved with manipulatives and diagrams before trying to model the situation using symbols or formal methods.[13] Routinely asking students to identify the quantities that are constant, invariant, or that covary, and describing how the relationships between quantities change using descriptive language, as well as encouraging them to model relationships mentally and draw diagrams, is important.[14] Allowing students to see, in a sustained way, proportional relationships across varied contexts such as slope, scale, probabilities, vectors and rates can be particularly effective.[8,10,14]

IMPLICATIONS: Students should have opportunities to represent proportion problems and quantities in informal, invented ways before moving towards symbols and algebra

Giving students frequent opportunities to describe and sketch quantities that change together in different ways is an important feature of proportional reasoning

Students should be encouraged to explore different areas of mathematics (including slope, scale, probabilities, vectors and rates) through a lens of proportional reasoning, seeing the connections and similarities

> "Multiplicative thinking is central not only in mathematics but in the application of mathematics in employment and everyday life, especially using percentages and proportions"[1]
>
> **Hodgen et al, 2014**

> "To fully develop children's number sense we need to make sure that they get a rich diet of operating with numbers as relationships as well as quantities"[2]
>
> **Askew, 2011**

Lucy Rycroft-Smith, Darren Macey, Rachael Horsman & Tabitha Gould, 2020

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