ISSUE 45 November 2022

Espresso

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CAMBRIDGE Mathematics

Lucy Rycroft-Smith, 2022

Talking point

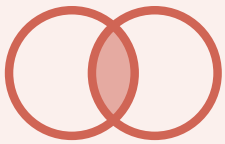
What does research suggest about the teaching and learning of early logical reasoning?

In summary

- Logic and logical reasoning are based on asking what is true and false and are central to many subjects including language, science, computer science and mathematics
- Logical reasoning is the basis for student understanding of concepts such as counting, equivalence and classifying
- Logical reasoning is making any argument using facts and connections, not just formal reasoning and proof; it can be analogical, abductive, inductive or deductive
- Understanding the difference between logic and belief and the way they interact supports students to develop logical reasoning
- Asking young students to complete idea generation tasks may help them to reason logically as it promotes creative and divergent thinking
- Exploring "all, none, some" type tasks and logic puzzles/games, as well as using both formal and visual representations, helps students to develop logical reasoning
- Giving students logical reasoning tasks with imaginary or nonsensical rules (false premises) helps them to develop abstract reasoning
- Possible tasks to support the development of early logical reasoning include "all x have y" types (implication), "knight and knave" puzzles, and exploring the use of "always" and "never" statements

Types of logical reasoning

analogical reasoning



using relational patterns by noticing similarities

saying "this thing is like that thing in this way"

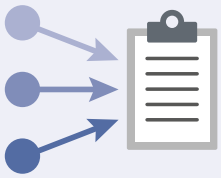
noting the ways things correspond between the structures of one system and another system

abductive reasoning

looking at the facts or situations before you and creating "most likely in the face of the facts" explanations

using reasoning such as Occam's razor (when choosing between possible explanations, choose the simplest one with the fewest assumptions)

inductive reasoning



starting from individual observations and drawing generalisations

making and testing conjectures

moving from the specific to the general

deductive reasoning

starting from accepted or agreed statements and using logical arguments to draw valid conclusions

constructing chains of reasoning using logical consequences

1

Logic is "the study of truth-preserving arguments"[1(p88)] often beginning with questioning what truth is. One of the most important goals of logical reasoning is to determine whether statements, and the arguments built from them, are true or false,[2] which cuts across many science, technology and arts subjects[3] as logic and argument are interdisciplinary ideas – in particular bridging language and mathematics in important ways[(e.g.4)] – and are central to both science and computer science. There is a "causal link between logical reasoning and mathematical learning. Much of children's mathematical knowledge is based on their understanding of its underlying logic"[5(p147)] including early concepts such as counting, equivalence and classifying, yet very little research exists on logical reasoning in primary education. It has been suggested that explicit teaching of logical reasoning is often ignored, which is a problem if students are to understand the principles on which mathematical and scientific thinking are built.[6]

Implications:

Logic and logical reasoning are based on a questioning of truth

Logical reasoning is central to many subjects including language, science, computer science and mathematics

Logical reasoning is the basis for student understanding of counting, equivalence and classifying, but is not often researched and may not be taught explicitly

2 Logical reasoning has been described as the construction of a whole or part of an *argument* (either within or outside of mathematics, including statistics), which is characterised by "claim, data, and warrant, where a claim is a statement to be established, data include facts used to support the claim, and warrants show connections between data and claims."[7(p181)] However, logical reasoning does not refer to just formal reasoning (such as deduction and proof), but also to a broader definition including any form of thinking where starting ideas and relations between them are used in a rigorous manner to infer conclusions.[8] Logical reasoning defined in this way is "selecting and interpreting information from a given context, making connections and verifying and drawing conclusions based on provided and interpreted information and the associated rules and processes."[9(p1676)] There are four main types of logical reasoning or inference: analogical, abductive, inductive and deductive (see infographic),[7,10,11] which can be used in argument and justification in all areas of mathematics and statistics.

Implications:

Logical reasoning is the construction of an argument using data and facts, and making connections

Logical reasoning in mathematics and statistics does not just refer to formal reasoning and proof, but any form of reasoning where ideas are used to make inferences in a rigorous manner

There are four main types of logical reasoning in mathematics: analogical, abductive, inductive and deductive

3 Understanding the difference between logic and belief, and the interplay between them (often expressed as bias) is a crucial part of students' education.[6] One study with 3–5 year-olds found that when given an idea generation task beforehand (such as "How many ways can you think of to make a noise?"), the children performed better on a logical reasoning task than if they had a card sorting task first, suggesting that the ability to generate alternative ideas (creativity) is important for the very early development of logical reasoning.[12] It is also suggested that young students have the opportunity to explore problems involving use of "all, some, none" structures, as well as a variety of logic games and puzzles, to help develop logical reasoning skills.[13] Reasoning with false premises (constructing logically valid but nonsensical arguments, such as on an imaginary planet: "On Kronus, if a person morps then they will become pled") was found to improve abstract logical reasoning in 12–15 year-olds.[14] Student development of logical reasoning is supported by the use of both visual and formal representations.[8]

Implications:

Understanding the difference between logic and belief and the way they interact helps students develop logical reasoning

Giving young students an idea generation task first may help them to reason logically as it promotes creative thinking

Even young students can explore "all, none, some" type tasks and logic puzzles/games

Giving students logical reasoning tasks with imaginary or nonsensical rules (false premises) may help to develop abstract reasoning

Using both formal and visual representations helps students to develop logical reasoning

4 Even very young students (ages 3–5) can explore logical reasoning (implication) tasks with prompts, such as "Someone told me that all dogs have legs. A friend of mine has an animal with legs. Is it true that the animal is a dog? Why?"[12] Students can explore logic puzzles such as "knight and knave" problems where they consider situations in which they know that one person always lies and another always tells the truth.[13] Older students can begin to explore questions such as "When you add any two even numbers, your answer is always even. Is this statement true or false?" in a variety of ways.[15]

Implications:

Possible tasks to help support the development of early logical reasoning include "all x have y" types (implication)

Students of all ages can also explore "knight and knave" and other logical puzzles to support logical reasoning

Older students can begin to explore the use of "always" and "never" statements; for example, in the context of odd and even numbers

> "The argument about whether head or heart is more valuable has raged forever. Would you rather possess the clear rationality of the Greek god Apollo or the wild emotion of Dionysus? The cold logic of Mr. Spock from "Star Trek" or the messy humanity of Dr. McCoy? … [T]he two constitute a kind of false dichotomy because they're actually inseparable"
>
> McDermott-Murphy, 2022[16]

> "How often have I said to you that when you have eliminated the impossible, whatever remains, *however improbable*, must be the truth?"
>
> Conan Doyle, 1890/2022[17]

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