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Espresso

A sip of filtered research

CAMBRIDGE Mathematics

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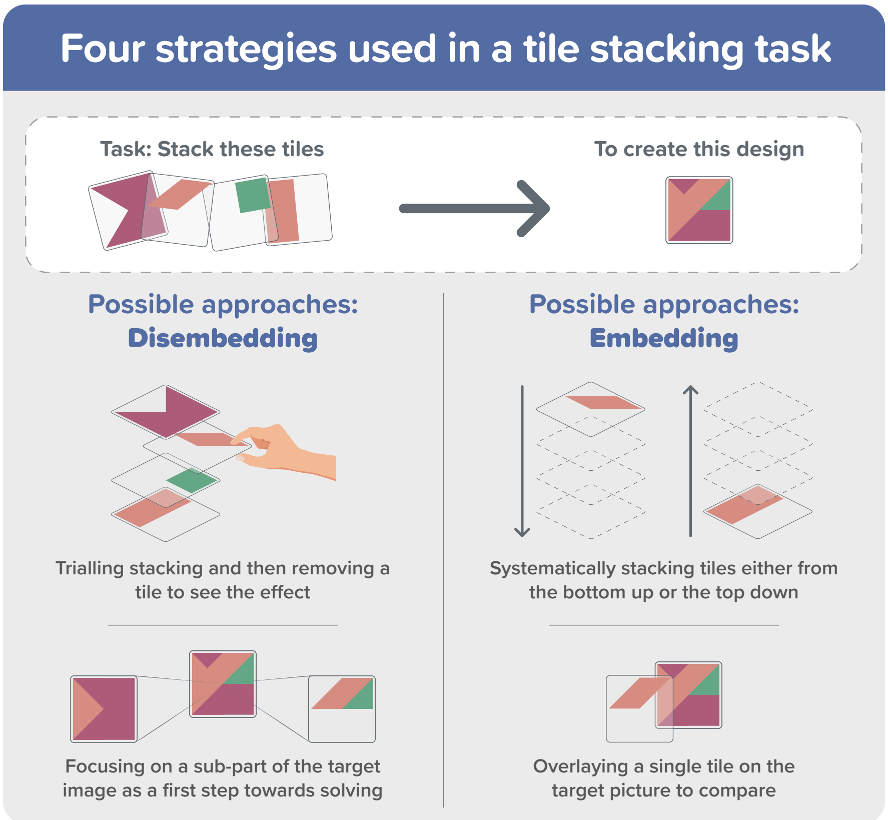
Talking point

What does research suggest about building up and breaking down 2D and 3D shapes?

- A general strategy of exploration, alongside building understanding and using specific vocabulary, is suggested to support understanding of and familiarity with 2D and 3D shapes
- Student experiences with shapes should include drawing, describing and playing with a range of straight and curved edged examples (not just polygons)
- Students can be encouraged to develop skills of composing and decomposing through hands-on experiences with shape that include exploration, trial and improvement, and intentional prediction and planning
- Useful student experiences that help develop fluent geometric composition and decomposition may include manipulating pattern blocks, making shapes, combining/ taking apart shapes and repeating combinations iteratively
- Providing activities that use overlapping shapes to make a target image (see infographic) can support students' embedding and disembedding; scaffolding these tasks to attend to colour, orientation and order at different stages may be helpful
- Constructing and deconstructing when building with 3D shapes develop skills that progress from random placements of shapes to planned construction which utilises their properties
- Structuring block play can support the development of spatial reasoning and scaled construction skills, and setting tasks within a storytelling context may increase engagement and spatial visualisation



Adapted from Bofferding et al., (2022).[1] Used under the Creative Commons Attribution (CC BY) license.

1 Building up and breaking down shapes is the basis for early geometric reasoning and includes manipulation of both 2D and 3D shapes. It encompasses embedding (overlapping or adding on to existing shapes, to form a new shape) and disembedding (identifying discrete shapes with another shape) and/or composing (combining shapes) and decomposing (breaking apart shapes). Students need time to play with different 2D shapes to understand many later geometric topics; students' understanding of ways to compose and decompose geometric shapes is foundational for developing later work related to the properties of shapes,[2] as well as reading graphs and exploring functions.[3] It is suggested that sequences of geometrical learning should begin with playful exploration, especially physical, and gradually develop conceptual understanding by incorporating related language and structures.[4]

Implications:

A general strategy of exploration, building understanding and specific vocabulary, and integrating experiences is suggested to support understanding of 2D (and also 3D) shapes

2 Polygons, although they are types of 2D shapes that students regularly meet across mathematics, are not consistently defined in mathematics education.[e.g., 5] Technical mathematical definitions often suggest that a polygon is a set of line segments plus the region they enclose, but given the common depiction of two-dimensional shapes as a perimeter with no shading, this can cause confusion as students may see them as just "frames" rather than filled shapes. This means that students could interpret "solid shape" to mean a filled/ shaded polygon as well as a three-dimensional shape, so careful attention to language is important.[6] It is recommended that students encounter and investigate a range of shapes (and not just focus on polygons) such as circles, ellipses (often known as ovals) and shapes with a combination of curved and straight edges.[7]

Implications:

Polygons are not well defined in mathematics education; spending some time investigating them, (for example, whether the word refers to the boundary or the area or both) may benefit students

Student experiences with shapes should include drawing, describing and playing with a range of straight and curved edged examples

3 Research suggests that there are many stages of composing and decomposing shapes;[8] for example, using pattern blocks (manipulatives of different geometrical shapes with angles that are multiples of 30º). These stages involve a (not-necessarily linear) progression that incorporates exploration, trial and improvement (experimenting with ideas and using the results to adjust the next try) and then intentional prediction and planning. The individual stages suggest that students gain experience with shapes in the following ways: manipulating, making pictures, combining/taking apart shapes, composing/decomposing (later with predictive intent, using angles, side lengths, rotation and flips), substituting and intentional repeating using ideas of units. The idea of putting together and taking apart is connected to important ideas across mathematics; for example, the ability to compose and decompose numbers.[8]

Implications:

Students can be encouraged to develop skills of composing and decomposing through playful experiences with shape that include exploration, trial and improvement, and intentional prediction and planning

Useful student experiences that help develop fluent geometric composition and decomposition may include manipulating pattern blocks, making shapes, combining/taking apart shapes and repeating combinations iteratively

4 A study of the strategies employed by students, aged approximately 6–7 and 8–9 years old, which was designed to clarify the factors that support students' embedding and disembedding, used two tasks from the puzzle game *Color Code* which involved stacking clear tiles with coloured shapes printed on them.[1(pp85–88)] The findings showed that the students in both age groups who were most successful used a combination of systematic disembedding (e.g., isolating parts of the design) and systematic embedding (e.g., testing tiles) – see the infographic. When embedding and disembedding, shape, colour, orientation and order (for solid stacked 2D shapes) are all attributes that can be attended to[9] and used to structure partial embeddings on the way to finding a solution. For example, different coloured shapes could be used initially, without consideration of their orientations or relations to other tiles, followed by making partial embeddings of sections of a design, before moving on to complex embeddings and those involving tiles of the same colour.[1]

Implications:

Providing games or activities that use overlapping shapes to make a target image (see infographic) can support students' embedding and disembedding

It may be helpful to scaffold tasks for students to help them attend to colour, orientation and order at different stages, before combining these aspects together in more complex activities

5 Stages of development associated with 3D shapes involve doing the following with building blocks: placing randomly; randomly stacking; placing in a line; creating planned stacks (using similar sized blocks); building a wall; building irregular/solid "rooms"; substituting component parts for one block; and creation of scaled and complex structures.[8] One study found that systematic teaching of block play amongst 5- and 6-year-olds is an effective way of developing spatial reasoning skills, and that designing mathematical content in a storytelling context further improves learning.[10] In addition to developing geometric and spatial skills, block play has been found to benefit wider mathematical learning and writing skills.[11]

Implications:

Constructing and deconstructing when building with 3D shapes can progress from random placements of shapes to planned construction which utilises their properties

Structuring block play can support the development of spatial reasoning and scaled construction skills, and setting tasks within a storytelling context may increase engagement and spatial visualisation

> "Seeing is not as simple as looking"
>
> **Kosuth, 1991**[12 (p57)]

> "There must be a good foundation of practical work ... Only in this way can the average [student] develop what I will call the geometrical "eye": the power of seeing geometrical properties detach themselves from a figure"
>
> **Godfrey, 1910**[13(p197)]

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