# Instant

A spoonful of subject knowledge

CAMBRIDGE Mathematics

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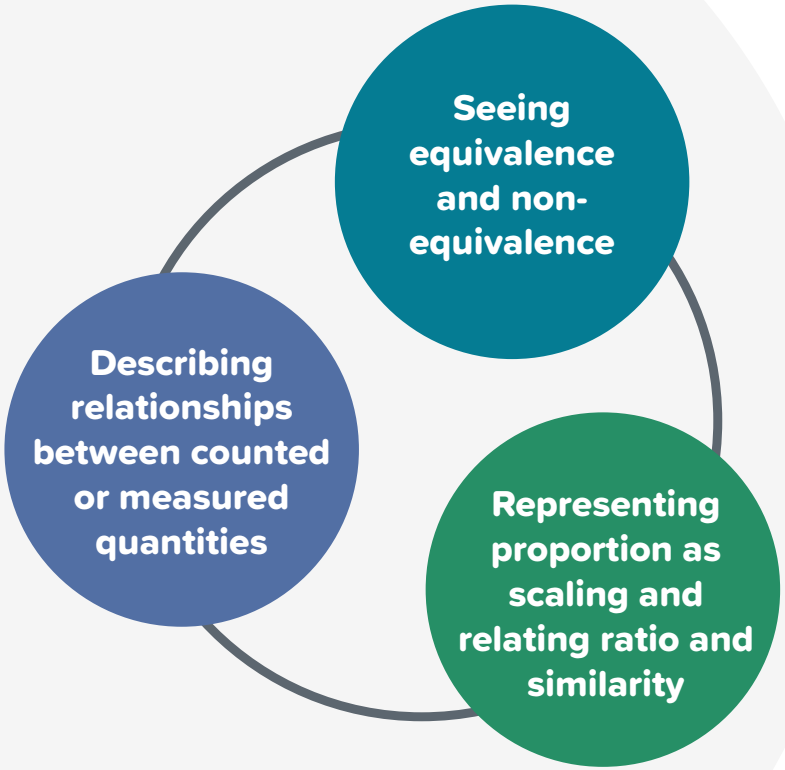
## Establishing concepts of ratio




In the beginning students explore ways to describe relationships between quantities. They begin to recognise that this may be done additively (as difference) or multiplicatively (as scaling, nesting, and/or ratio). They think about what the parts of a ratio might mean.

In the next stages students make sense of equivalent ratio relationships by making and talking about a variety of pictures and diagrams. They think about how this relates to other representations of proportion such as fractions, explicitly considering part-part, part-whole, and whole-whole models.

Making sense of these things together, students co-ordinate fraction, ratio, percentage and decimal representations of proportional relationships. They think about the ways ratio is embedded in concepts of similarity and may begin to think about ratio as a constant rate of change.

### Explore

- multiplicative comparison
- unit rate

### Key research source

Middleton, J. A., & van den Heuvel-Panhuizen, M. (1995). The ratio table. Mathematics *Teaching in the Middle School, 1*(4), 282–288.

**Short:** 8 or fewer pages long

**Earlier:** more than 16 years before Instant publication

**Key question: How do we know two ratios are equivalent?**

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# Establishing concepts of ratio

## Describing relationships between counted or measured quantities

- ‘How much bigger is this one?’
  ‘That one is three times as big’
- ‘How many more is that one?’
  ‘That one is six more’
- ‘I’ve got one cup of sugar, how many eggs do I need?’
  ‘I think you need two eggs’
- ‘How did you make your drink of orange squash?’
  ‘I filled my glass to here with squash, then topped it up with water’
- ‘Can you write that as a ratio?’
  ‘It’s 7 mm : 5 mm’
- ‘In this ratio, 1:6:5, what is the 6?’
  ‘It’s the number of blue counters’
- ‘Can I draw a rectangle where the height is three times the width?’
  ‘Yes, there are lots! 3 cm and 1 cm works, or 21 cm and 7 cm’
- ‘Can I draw a picture to show how much smaller this one is?’
  ‘I could line them up to show the extra bit’

## Seeing equivalence and non-equivalence

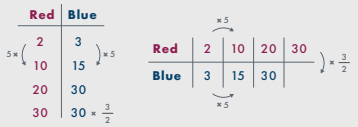
- ‘I’ve got two cups of sugar, how many eggs do I need?’
  ‘I need two eggs for every cup of sugar’
- ‘I want to make half as much’
  ‘That means I need to divide everything by two’
- ‘Which space is more crowded?’
  ‘This one’s smaller, but it’s also got fewer people in it’
- ‘I like the taste of your smoothie, how did you make it?’
  ‘I used three times as much milk as banana’
- ‘Is 3:9 the same as 4:12?’
  ‘Twelve is three times as much as four’
- ‘That person can run 100m in 12 seconds’
  ‘So can they run 1,000m in 120 seconds?’
- ‘How do I scale up this recipe?’
  ‘I need the same amount of flour, butter and sugar’

## Representing proportion as scaling and relating ratio and similarity

- ‘How do I scale up this pattern?’
  ‘A fifth of the picture is blue’
- ‘What’s the likelihood of picking green?’
  ‘Green and blue are in the ratio 4:1 so the probability is 4/5 or 80%’
- ‘What’s the likelihood of getting blue?’
  ‘It’s the same likelihood of landing on blue, whichever spinner you use’
- ‘Could I use a ratio to show that?’
  ‘I could also use a decimal, a fraction, or a percentage’
- ‘Is 4:3 the same as 4/3?’
  ‘Is 3 one part, or the whole amount?’
- ‘Are these two shapes similar?’
  ‘This pair of sides are in the same ratio’
- ‘Which is faster, 30 m : 6 s or 40 m : 7 s?’
  ‘I need to make something the same’
- ‘When you divide the lengths by the widths you get the same number’
  ’The diagonals of these rectangles line up’

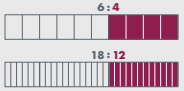
## Models and tools

### Ratio tables



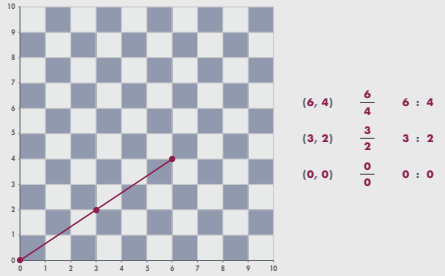

### Double number lines


### Bar models




### Spinners

### Grids and graphs



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