CAMBRIDGE Mathematics

Case study micro-report

Mapping MathemaTIC tasks to the Cambridge Mathematics Framework

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One of the uses for which we are designing the Cambridge Mathematics (CM) Framework is mapping mathematical tasks to the mathematical ideas which form the core of the CM Framework's content. We worked with Vretta, a company which has developed a personalised assessment-for-learning platform called MathemaTIC (in collaboration with education partners) that contains a database of formative and summative mathematical tasks, to implement the CM Framework in a task mapping exercise. Vretta mapped MathemaTIC tasks from two separate middle school mathematics topics to two collections of content in the CM Framework and filled out a post-task self-report questionnaire. They found some of our finer-grained content descriptions to be very useful for task mapping and had suggestions for additional content which could be helpful. The exercise yielded insights for both Vretta and the Cambridge Mathematics team and is presented here as an example of possible task mapping processes and design outcomes.

Introduction

We present the background for the series of case study micro-reports in a separate paper[1]. This particular case study micro-report presents the MathemaTIC task mapping case study. The results and discussion offer an example of what can be learned from mapping a set of tasks to the CM Framework.

We define *mapping to the CM Framework* as creating a link which indicates a correspondence between an element of external mathematical content and an element of mathematical content within the CM Framework. We use an open-ended definition of a task as "a problem…that focuses student attention on a mathematical idea" (Stein, Grover, & Henningsen, 1996, in Nyman, 2016, p. 1510). We are exploring the connections which can be made between the CM Framework and tasks because of the crucial role played by mathematical tasks in how the enacted curriculum plays out in the classroom: "the tasks used in mathematics classrooms highly influence the kinds of thinking processes in which students engage, which, in turn, influences student learning outcomes" (Stein, Grover, & Henningsen, 1996, p. 462).

[1] Background for the case study micro-report series (Jameson, 2019)

Case context: task alignment actions and outcomes with Vretta

Vretta is a Canadian EdTech company which develops formative assessment tasks for a variety of international contexts in mathematics education and delivers them through digital learning and assessment platforms. The Vretta team was interested to discover what insights they might gain from aligning some of their MathemaTIC tasks with the Cambridge Mathematics Framework. Initial possibilities, from their perspective, included

- examining implied differences between their current expected sequencing of mathematical experiences and the ordering represented in the CM Framework,
- finding gaps in the ideas their tasks targeted, suggesting that new tasks could be developed to provide students and teachers with helpful formative feedback, and
- finding gaps in our coverage, using their implementation data to point out connections or ideas we might consider including in the CM Framework.

This task alignment case involved the use of the main set of CM Framework visualisation tools and content in two selected topic areas. This would be the first time the CM Framework tools and content had been accessed directly by an external team; the Cambridge Mathematics team was therefore interested to know

- what the Vretta team's goals were as users of the CM Framework,
- whether they were able to achieve these goals,
- what actions they were able to take, or wanted to take but couldn't, and
- how these goals and actions might inform the expression of our design principles in the CM Framework: specifically what forms of information represented in the CM Framework seemed most meaningful for identifying, ordering or proposing tasks.

Furthermore, based on reviews of existing knowledge maps, we would expect those which are useful and usable in a case like this one to support users in doing some specific actions indicated for this case in Table 1. Five are indicated as being particularly relevant to the MathemaTIC task mapping scenario.

Table 1: Expected affordances of knowledge maps; those of particular relevance to the MathemaTIC task mapping case are indicated under 'This case.'

	Combined user groups	This case
Help designers to communicate ideas about knowledge to others; make tacit ideas explicit and present ideas in a form that users can relate to (Eppler, 2004; Vail, 1999)	✓	✓
Help users to "remember, comprehend, and relate knowledge domains through insightful visualization and aggregation of information" (Eppler, 2004, p. 200)	✓	
"[M]ake information actionable in new contexts, connect it with previous experiences" (Eppler, 2004, p. 189) – that is, professional learning and transfer	✓	
Help users to evaluate what knowledge is available for decision-making and from what sources (Eppler, 2004)	✓	✓
Help users to see concepts within a bigger picture and to switch between multiple perspectives (Eppler, 2004)	✓	✓
Help users to evaluate and compare sets within knowledge domains – examining what knowledge is available, from what sources, and with what justification (Eppler, 2004)	✓	
Provide a "common framework" when searching for or contributing "relevant knowledge" (Eppler, 2004, p. 190), which itself supports professional learning	✓	
Contribute to the field by providing a big-picture perspective and a research base with respect to ideas that people in different roles may hold in common	✓	
Relate the big-picture perspective to different levels of underlying detail (Eppler, 2004)	✓	✓
Support professional learning in practical contexts: "just-in-time" (Vail, 1999, p. 23)	✓	✓

Role of the MathemaTIC task mapping study within our evaluation plan

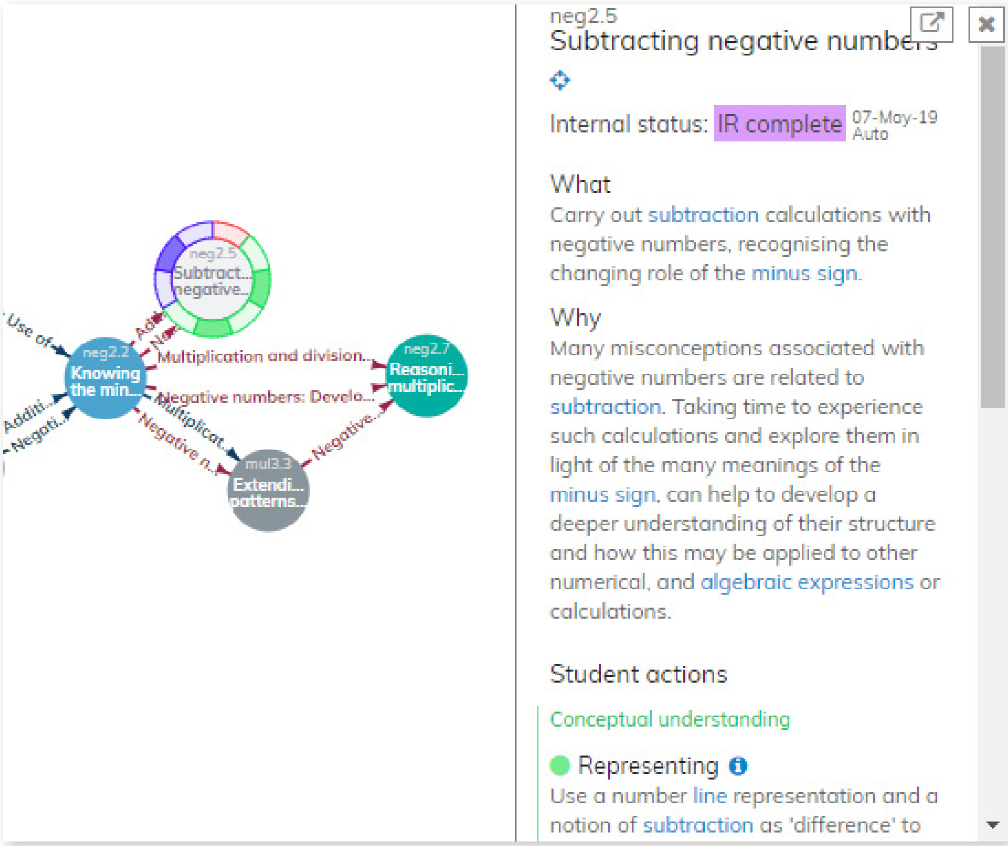
The MathemaTIC task mapping study was timed to inform discussions about the suitability of knowledge maps generated from the CM Framework for educational designers, particularly task designers, to use in their work. This has helped us to understand needs for task layer features for end users prior to developing them.

Methods and materials

Cambridge Mathematics Framework structure and content

In the Cambridge Mathematics Framework, mathematical ideas and key relationships between them are represented in a network of *waypoints*, which we define as "places where learners acquire knowledge, familiarity or expertise" (Jameson et al., 2019, p. 4). Waypoints have titles, descriptions (*what* we mean by it and *why* it is there) and *student actions*, which are examples of the kinds of things students might do to help them build an understanding of the content at a waypoint. An example of a waypoint and its content is shown in Figure 1. We call the relationships between waypoints *themes*, and in the CM Framework they represent either the development of some part of an idea from one waypoint to the next, or the use of one idea contributing to the understanding of another[2].

Figure 1: Example waypoint with content ("what," "why" and "student actions")



[2] Waypoints, themes, and types of student actions are described in Ontology: Structure and meaning in the Cambridge Mathematics Framework (Jameson et al., 2019)

Research Summaries are documents in which a subset of this network of waypoints and relationships is embedded; they "tell the story" of the content and structure of a set of waypoints and themes. Each Research Summary includes three potential elements of interest for this case: (1) a review of the literature informing the content, (2) the embedded CM Framework content and (3) a description of how our interpretation of the sources in our research base has led to the structure shown in that Research Summary. We determined that Research Summaries would be the most accessible way, at our current stage of interface development, to present CM Framework content for external use.

Research Summaries can be viewed as static documents or through the online CM Framework platform, CMF Nexus (Stevens et al., 2019). In CMF Nexus, the embedded content can be viewed dynamically so that the content for each waypoint can be viewed in full one waypoint at a time, or it can be output as a spreadsheet for users to scan across waypoint content in a different way.

MathemaTIC items and tasks

Vretta positioned their work with the CM Framework within a database of formative assessment task items aligned with the MathemaTIC curriculum. Vretta, along with their partners and collaborating institutions[3], developed the MathemaTIC task items. The tasks have been developed in line with a framework of task attributes which Vretta created to structure their work. This framework specifies 31 types of interactive tools which can be embedded within tasks (e.g. decomposition line, polygon draw tool, etc.), 15 types of answers which can be submitted (e.g. drag graph, multiple choice), and 14 types of formative feedback students can receive as they work through tasks in the MathemaTIC learning environment.

The MathemaTIC platform typically presents collections of tasks as *items*, which students then encounter (though occasionally an item might consist of a single task). The MathemaTIC task framework distinguishes between three item types: *Lessons* for teaching (blue text in Fig. 2 labels), *exercises* for practice (red text in Fig. 2 labels) and *challenges* to extend learning (black text in Fig. 2 labels). Some collections of items serve as diagnostic or summative *tests* (grey text in Fig. 3 labels).

[3] in partnership with The Ministry of National Education, Children and Youth of the Grand Duchy of Luxembourg and in collaboration with the National Ministry of Education in France, the Luxembourg Centre for Educational Testing, the Luxembourg Institute of Socio-economic Research, the University of Luxembourg, and CGIE. This partnership and collaboration, since 2015, is ongoing for the continued expansion of the MathemaTIC curriculum and item tasks for primary and secondary school students

The Vretta team had previously aligned their items with the MathemaTIC curriculum. This meant that prior to their work with the CM Framework they had made a set of decisions about ordering and dependencies of mathematical ideas in that particular curriculum. Both teams were interested in how the content and structure of the CM Framework might interact with those previous decisions; whether it might suggest something additional or different.

Vretta team

The members of the Vretta team engaged in these mapping activities had similar backgrounds to the members of the Cambridge Mathematics team writing the CM Framework: they had previously been teachers and had studied mathematics at university level before that. Their current roles were in task and assessment design. They had also been involved in authoring the items they were mapping and used their knowledge of the item designers' pedagogical intentions as well as the content of the items to do the mapping.

Determining user goals, outputs and ways of working

At the initial meeting, the Cambridge Mathematics team and the Vretta team demonstrated their tools, content and ways of working, and discussed their respective goals for the case study. The Vretta team then provided the Cambridge Mathematics team with overviews of their tool, answer and feedback types. The Cambridge Mathematics team compiled a list of completed Research Summaries with topics overlapping the MathemaTIC database of task items and provided it to Vretta. The Vretta team selected the *Negative numbers* and *Early Area Calculations* Research Summaries to analyse how their content in these areas aligns with current research.

The pilot case protocol

The Cambridge Mathematics team developed components of a protocol to support the Vretta team's use of the CM Framework. We also elaborated on our guiding questions to create a post-mapping self-report questionnaire which could help structure feedback according to the specific interests of the Cambridge Mathematics team. The package developed for Vretta's activities consisted of four documents:

1. A two-page summary outlining the purpose and scope of the activities Vretta would be undertaking, the role of case study feedback in the Cambridge Mathematics design process and particular focal points specified by the Cambridge Mathematics team
2. A two-page quick-reference sheet summarising the meaning of each feature in the CM Framework[4]
3. A questionnaire delivered through SurveyMonkey containing the questions described in (1)
4. A consent form outlining the purpose of the research, how data would be collected and stored and how it might be reported.

The documentation was minimal because one of our goals was to see which ways of working with this content would be most useful to the Vretta team, including any processes they might develop on their own.

We agreed with Vretta that their main activity as CM Framework users would be mapping their items to features in the CM Framework within the topic areas of the two Research Summaries we provided. The resulting links were to be recorded outside the CMF Nexus environment so that it would not be necessary for the Vretta team to learn to use the Cambridge Mathematics team's design and authoring tools. Vretta would then report their experiences in the questionnaire and meet with Cambridge Mathematics to discuss the results.

User post-mapping self-report questionnaire

Amongst other things, we asked the Vretta team members about their role(s) and background, how they chose tasks to map to our CM Framework, how they mapped them (i.e. what level(s) of information seemed relevant), whether this mapping might make sense at the level of activities or groups of tasks, and whether some of the structural roles we identified for certain waypoints in the CM Framework seemed related to any particular type of task. The full set of items in the questionnaire is given in Appendix A.

The Vretta team designated a primary CM Framework user to respond to our questionnaire, but they discussed and refined the answers beforehand as a team based on their collective experiences using the CM Framework. They also developed a set of shared documents in which they did their mapping activities and provided those to us for the debriefing discussion along with their responses to the questionnaire.

[4] See Appendix A and Ontology… for more information

Debriefing

We followed up the self-report questionnaire with another in-person meeting for clarification and follow-up questions, involving representatives from Cambridge Mathematics' software design and development and writing team. This meeting and subsequent team discussion served to decide on the outcomes and disseminate them to the rest of the team.

The Cambridge Mathematics team sent Vretta the package of documents, the questionnaire, and accounts and instructions for viewing content within the *Negative numbers* and *Representing quantitative data* Research Summaries in CMF Nexus.

Time frame

The time frame for the activities described here extended from July 2018 to June 2019. The Vretta team had access to selected portions of the CM Framework and the questionnaire for a period of four months before completing their self-report, but only a portion of that time was spent on pilot case activities. The final debriefing discussion occurred a month after the questionnaire was completed.

Results

For each Research Summary, we report results in terms of item-mapping *actions*, *artifacts* and *interpretation*. *Actions* are what the Vretta team did, with their content and with CM Framework content, in order to map items from their database to waypoints in each Research Summary. *Artifacts* are shareable things (maps, lists, tables) produced either as intermediate aides in mapping or as a final output by either the Vretta or the Cambridge Mathematics teams. Forms of *interpretation* of the task mapping included

- observations of alignment or discrepancy between the sets of items and waypoints (how much they seem to "overlap"),
- observations about the relative structure and complexity of items and waypoints,
- observations about the "location" in the waypoint map, from left to right, of mapped items, and
- observations about the nature of ideas represented in the task item set relative to the student actions they are mapped to.

The Vretta team's interpretation provided qualitative interpretation in the self-report questionnaire and the Cambridge Mathematics team created frequency tables to explore some additional details relevant to CM Framework content.

Negative numbers Research Summary: Mapping tasks to CM Framework features

The Vretta team began with an initial set of items from their *Grade 7/8 Integers* module which they believed could be related to the *Negative numbers* Research Summary content. They read through the Research Summary and looked at the embedded waypoint content, which included descriptions and student actions for each waypoint and names for each connecting theme. They also read through our self-report questionnaire (see Appendix A for a list of items).

They created a diagram mapping each waypoint from *Negative numbers* with a list of their corresponding task items, if applicable (see Figure 2). Based on our mention of student actions in the questionnaire, they also created a spreadsheet containing the content of all CM Framework waypoints in the Research Summary to help them focus on all of the student actions (SAs). Task items which seemed to correspond well with particular SAs were listed next to them.

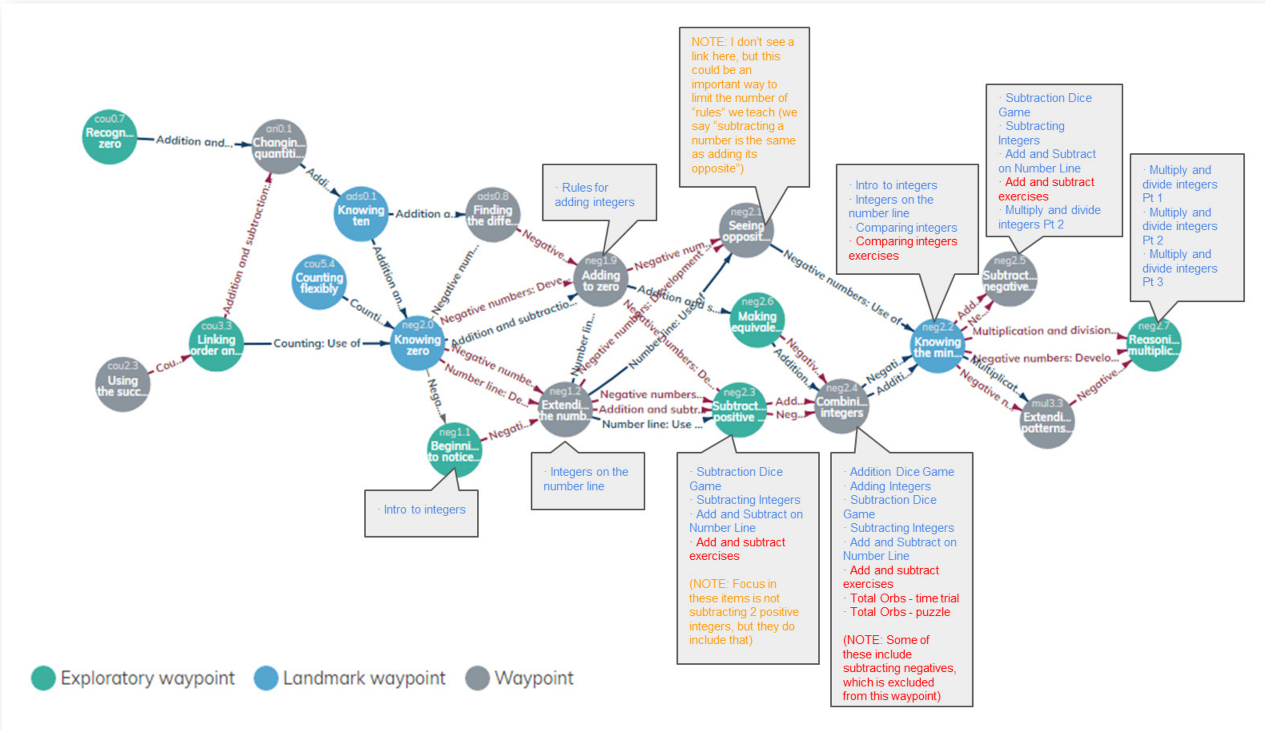
Out of 19 waypoints in the Research Summary, 8 waypoints had task items mapped to them, as shown in Figure 2. Of 16 mapped MathemaTIC items, 75% were lesson items, while 25% were exercises. The set of waypoints in Fig. 2 shows a sequence of conceptual development from left to right; the lesson items were mapped onto waypoints throughout this sequence, while the exercise items began to be mapped onto waypoints halfway through it, towards the right end of the sequence. The later waypoints had a mix of item types mapped to them, while the earlier half were only mapped to lesson items. The earliest waypoints in the Research Summary (connected by the CM Framework's 'counting' and 'addition and subtraction' themes) had no items mapped to them; this was mainly due to the fact that the Vretta team was drawing on tasks designed for 7th and 8th graders which did not focus on early counting, addition and subtraction. No challenge items were mapped.

Waypoints were mapped to between 1 and 8 items, with counts shown in Table 2. There was no single "size" of waypoint in terms of the number of items mapped to it, but half of the waypoints had 3 or fewer items mapped to them.

Table 2: Occurrence of waypoints by number of MathemaTIC items mapped to them (Negative numbers)

Number of items	1	2	3	4	5	6	7	8
Number of waypoints with that number of items mapped	3	0	1	2	1	0	0	1

Figure 2: Diagram mapping MathemaTIC tasks to CM Framework waypoints (Negative numbers)



A breakdown of items mapped directly to student actions (SAs) is given in Table 3. Of the items which were mapped to SAs, 4 items were mapped to two different types of SAs and 8 items were mapped to a single type of SA. 4 tasks were not mapped directly to SAs. The cases in which an item was mapped to two different types of SAs prompted the Vretta team to consider whether the item was focused enough for its purpose.

Table 3: Task items mapped to student actions in the Negative numbers Research Summary

Student action	Classifying	Representing	Arguing	Solving
Total mapped tasks	4	10	2	3

Table 4: Student action mapping count: Number of SA-mapped task items by number of SAs in the Negative numbers Research Summary

Number of student actions	0	1	2	3
Number of items mapped to student actions	4	7	3	2*

* One on a single waypoint, one on three different waypoints

Table 4 shows the number of items mapped to multiple, single or no student actions. 69% of the items were mapped to one or none of the SAs, while the rest were mapped to multiple SAs which were sometimes but not always on the same waypoint.

In the questionnaire, the Vretta team reported that the process of mapping their items to waypoints and student actions in the *Negative numbers* Research Summary was inspiring and challenging. It gave them an understanding of the coverage of items, a focus of the relevant pool of items and ideas for new items. They said that interpreting waypoint content was clear and that it was helpful that waypoints offered multiple ways to understand the nature of the mathematical ideas they refer to. In particular, they described student actions as being the most useful form of presenting these ideas for their purposes.

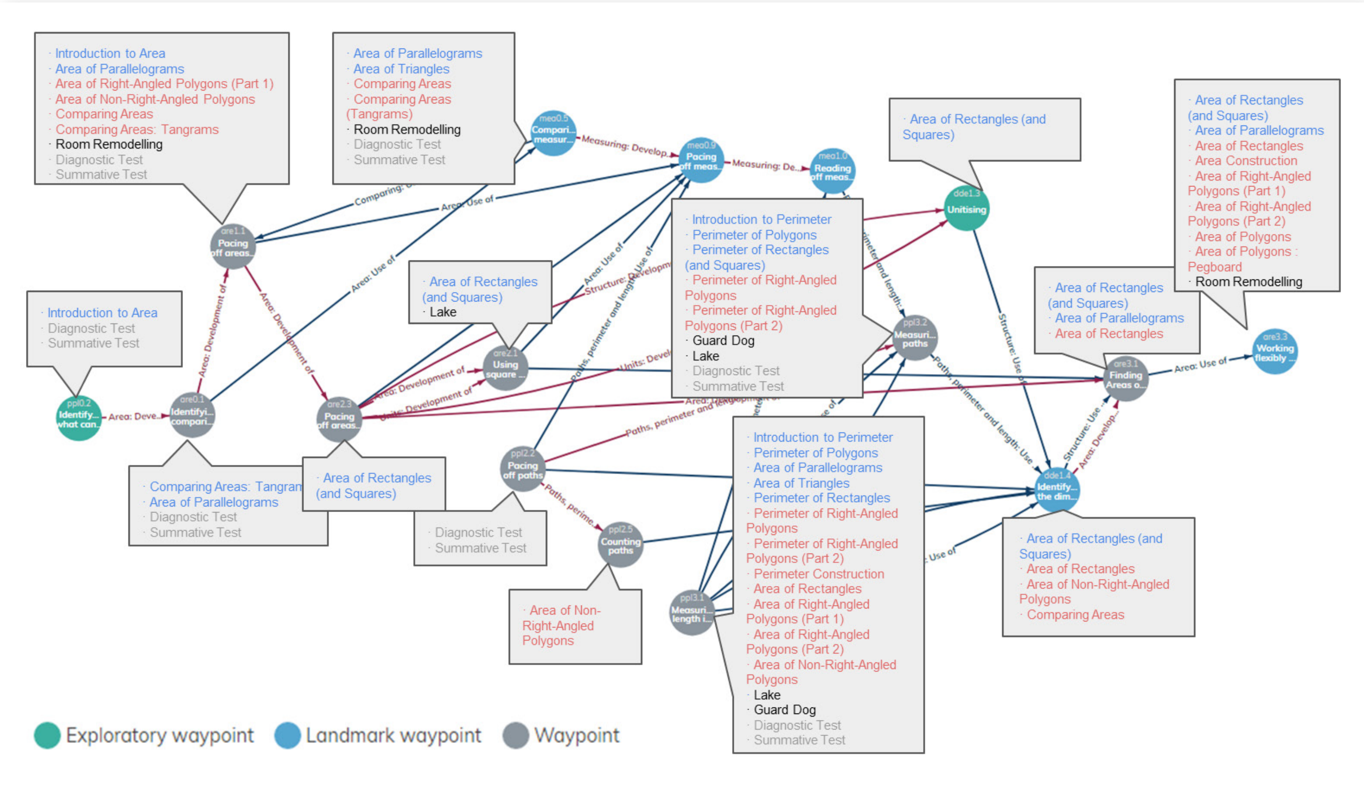
Early area calculations Research Summary: Mapping tasks to CM Framework features

The Vretta team began with an initial set of items from their *Cycle 4 Area and Perimeter* module which they believed could be related to the *Early area calculations* Research Summary content. As before, they read through the Research Summary and looked at the embedded waypoint content, which included descriptions and SAs for each waypoint and names for each connecting theme. They constructed a spreadsheet of waypoint content including SAs and items mapped to specific SAs when appropriate.

A total of 22 MathemaTIC items were mapped to 13 out of the 16 waypoints in the *Early area calculations* Research Summary, as shown in Figure 3. Eight waypoints were mapped with a mix of the three item types (lesson, exercise and challenge), four waypoints were mapped solely with lesson items, and one solely with an exercise item. Vretta treated both diagnostic and summative tests as item types and mapped them to 5 waypoints. We are not considering diagnostic and summative tests as items in our item counts below (Tables 4-7) because we presume they consist of a number of individual items with topics which are not specified here; nevertheless half of the mapped waypoints were mapped to them.

Four of the five waypoints with only lesson items mapped to them came in the ‘earlier’ (leftmost) half of the set of waypoints.

Figure 3: Diagram of mapped MathemaTIC items to CM Framework waypoints (Early area calculations)



Waypoints were mapped with between 1 and 14 items, with counts shown in Table 5. There was no single ‘size’ of waypoint in terms of the number of items related to it; the distribution was wide, though it was still more common for fewer items to be mapped to any given waypoint.

*Table 5: Occurrence of waypoints by number of task items mapped to them (Early area calculations)**

Number of items	1	2	3	4	5	6	7	8	9	→	14
Number of waypoints with that number of items mapped	4	2	1	1	1	0	2	0	1		1

* Diagnostic and summative test items not included

The Vretta team also made 68 mappings directly between items and CM Framework SAs. Fifty percent of these items were mapped to the Performing action, and 15% to solving, with 7 out of the 9 available SA types mapped on (Table 6). Modelling and Critiquing were the only SA types not mapped, but of these, Modelling was only an option on one of the waypoints, and Critiquing was not represented within the waypoints in this Research Summary.

*Table 6: Task items mapped to Student Actions in the Early area calculations Research Summary**

Student action	Performing	Classifying	Representing	Analysing	Arguing	Estimating	Solving
Total mapped tasks	34	6	4	5	4	5	10

* Diagnostic and summative test items not included

*Table 7: Student action mapping count: Number of SA-mapped task items by number of SAs in the Early area calculations Research Summary**

Number of student actions	1	2	3	4	5	6	→	10
Number of items mapped to student actions	5	8	2	4	1	2		1

* Diagnostic and summative test items not included

Table 7 shows the number of items mapped to multiple, single or no SAs. For this research summary, up to 10 items were mapped to a single SA, though on balance items still tended to be mapped to one or two SAs (59% of the 22 tasks).

45% of the 22 tasks were only mapped to a single SA type, with 50% mapped to two or three task types, and 9% mapped to four or five types (Table 8).

*Table 8: Mapped student action types count: Number of SA- mapped task items by number of different types of SAs mapped **

Number of student action types	1	2	3	4	5
Number of items mapped to student actions	10	6	5	1	1

* Diagnostic and summative test items not included

On the questionnaire, the Vretta team reported that the student actions were still very useful to them for item mapping. However, in contrast to the *Negative numbers* content, the Vretta team reported not feeling as “in tune” with *Early area calculations* content. They felt there was more overlap between the ideas in the waypoints which made them seem less “distinct” and which in turn “made the different perspectives feel less enlightening than last time” (the waypoint “whats”, “whys” and SAs).

Comparing mapping in both Research Summaries

The *Early area calculations* Research Summary contained more waypoints, and Vretta mapped more items to it than they had in *Negative numbers*. For both Research Summaries, more items were mapped to waypoints which came “later” in the sequence of conceptual development (more to the right in the map). In general, while items tended to correspond to fewer numbers of waypoints, there were some in each Research Summary which corresponded to many; both MathemaTIC items and waypoints are not all the same “size” or level of complexity in terms of the ideas which may be involved in engaging with them.

At first glance, there seemed to be differences between MathemaTIC’s two sets of items according to how they mapped on to student actions in the two Research Summaries. It looked as though MathemaTIC items mapped on to student actions within the *Negative numbers* waypoints were more aligned with “representing” actions, while items mapped on to student actions within the *Early area calculations* waypoints were more aligned with “performing” actions. It also seemed that the items mapped on to student actions within the *Early area calculations* waypoints represented a broader selection of actions. However, it was not clear simply from looking at the item mappings whether this was because of differences between the two sets of MathemaTIC items or between the types of student actions available in each Research Summary. The discrepancy between the types of student actions mapped to each set of MathemaTIC items and the pool of available student actions is shown in Fig. 4.

Figure 4: Differences between the proportion of item-mapped student actions to the proportion of available student actions in each category

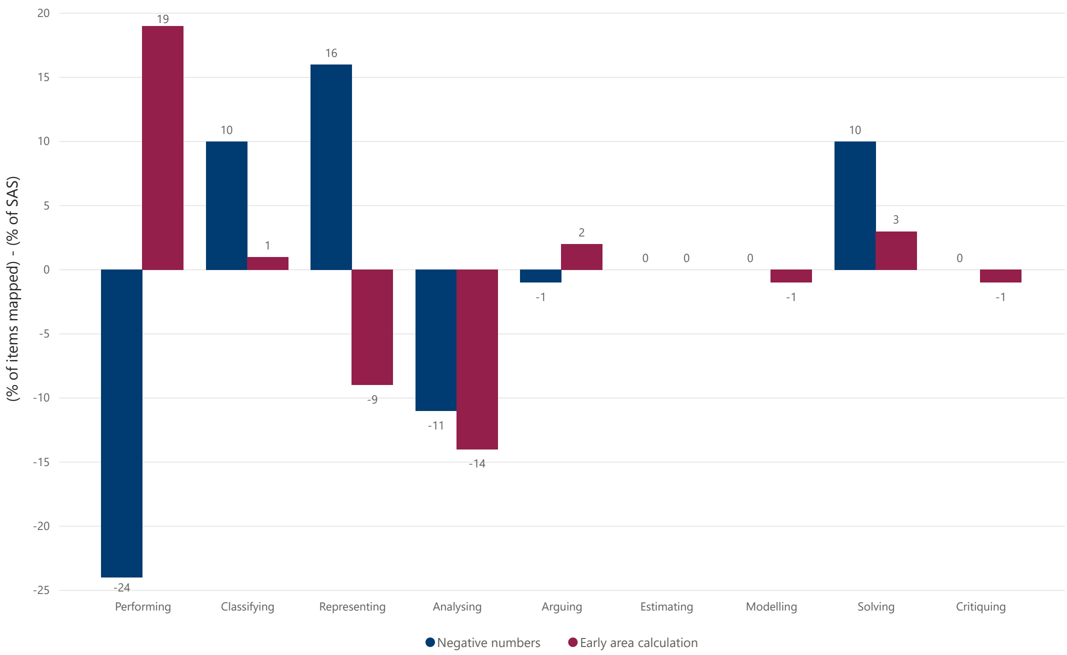


Figure 4 shows that both MathemaTIC item sets involved less analysing, modelling and critiquing than the SAs could have provided for, but mapped more heavily to solving and classifying. Larger differences between the item sets for each Research Summary were found in mappings to "performing" and "arguing" actions (fewer than available for *Negative numbers* mappings, more for *Early area calculation*), and to "representing" actions (more than available for *Negative numbers* mappings, fewer for *Early area calculation*).

The CM Framework content is still under development, so in some cases glossary terms were indicated in the waypoints but no definitions were available. The Vretta team reported that these glossary definitions would be very helpful, particularly since they noted some unfamiliar terms in use which were likely to be specific to the cultural context of the CM Framework writers. Apart from that, they didn't report that anything else was overtly missing; however their experiences across Research Summaries suggested to them that mapping was easier in cases with less seeming overlap between the mathematical ideas in different waypoints.

The Vretta team described CM Framework waypoints as being "clearly made up of several concepts" and described their own items as being "usually collections of tasks". They said that they felt they were able to map successfully because they had knowledge of the tasks which made up their items, and could see examples from the student actions of the ideas which made up waypoints in the CM Framework. However they said that extending fine-grained mapping (as they did with student actions) to the task level within their own items might make mapping too complicated. The mapping exercise suggested two additional uses which could be relevant to them in the future: item creation and linking with assessment questions for "remediation" or targeted feedback and practice.

Discussion

This case has given us some examples of what kinds of things can be learned when mapping a set of task items to the CM Framework, as well as insight into the process itself which can inform the support we are able to provide for that process. Both the Vretta team and the CM Framework team gained insights into their own content and ways of working as a result of Vretta's mapping activities. The activities reported to us also suggest that the expected benefits of the CM Framework as a knowledge map (see Table 1) were realised in this case.

Based on the Vretta team's questionnaire responses, there may be a difference in the way mathematical ideas are presented in the two Research Summaries which affects item mapping. This suggests that mapping activities might be of use to CM Framework writers in their reflective practice; possibly to increase consistency, or to explain such differences in terms of the underlying research upon which they are based. The Vretta team also indicated that they felt mapping could be useful for task item design, both for checking the coverage and focus of a set of items and for improving feedback for items integrated into a system for formative and summative assessment. Student actions turned out to be important features of CM Framework content for this mapping exercise and for the potential uses of mapping task items more generally.

The main user action for task-mapping which has emerged from this case is something we will call *switching frames*. The Vretta team reported switching between framing mathematical ideas according to their item database, the CM Framework waypoints map, waypoint-level content (descriptions and student actions) and, to a lesser degree, the Research Summary text in order to determine the correspondence they wished to highlight between their task items and CM Framework waypoints and student actions. While the use of the CM Framework for textbook authoring or curriculum refinement involves *zooming in and out* (switching between mathematical topics like Research Summaries and the big picture), and *tracing backwards and forwards* (switching between a hypothetical student's past, present and future conceptual repertoires), "switching *frames*" does not necessarily involve a shift in scale or topic. Rather, it involves looking across different forms of description, representation and/or instantiation of a relatively small set of mathematical ideas to link them appropriately out of a curriculum framework and into an applied context.

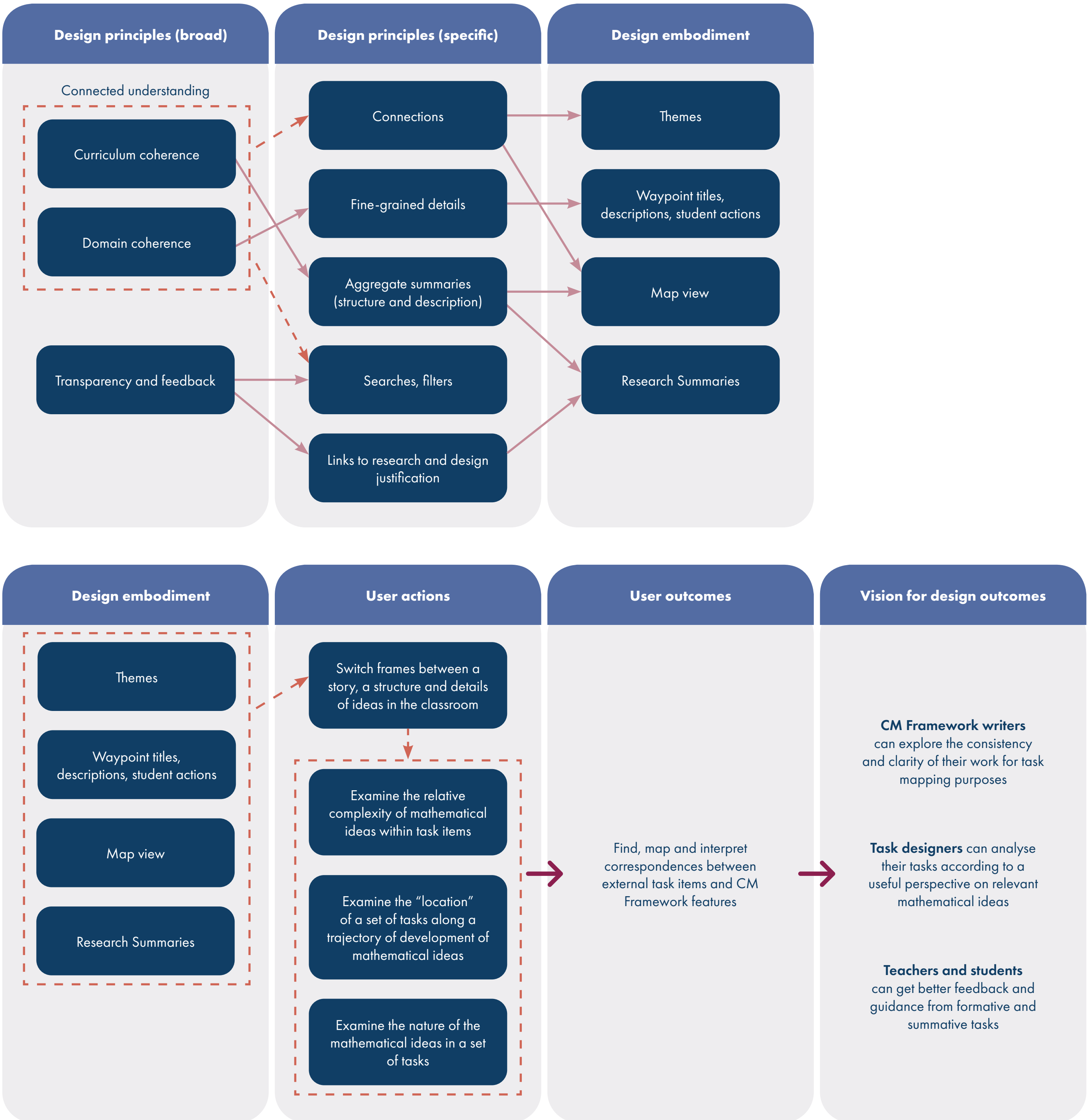
The "switching frames" mapping action depends on the following elements of the design of the CM Framework in particular:

- Theme edges and map visualisations show connections
- Waypoints with their titles, descriptions and student actions give detail
- Research Summaries provide a narrative explanation of how the research has informed the creation and structuring of waypoint content in the map.

These in turn can be traced back to specific interpretations of some of the broader design principles shaping the design of the CM Framework. A model linking our design principles to features in the design of the CM Framework and the user actions and outcomes they enable is shown in Figure 5.

Figure 5 on next page

Figure 5: Case-specific logic model of CM Framework design focusing on the "switching frames" user action



Keeping track of what the CM Framework design enables people to do, and how and why, helps us to evaluate the progress of our design work and structure additional formative evaluation to assist with creating interfaces and visualisations necessary to support key uses[5]. This task mapping case will contribute to a related case study exploring curriculum mapping actions, and to more targeted formative evaluation of interfaces and tools as development continues. This initial experience suggests that mapping a task to a reference framework like the CM Framework has the potential to be valuable in several ways and can be readily achieved using elements which are already part of the CM Framework design.

[5] Described further in Methodology: Research-informed design

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Appendix A: Protocol package document 1

Recording Experiences with *CMF Nexus*

Using *CMF Nexus* to map assessment and task items

> *"Nexus of meaning" (Sinnzusammenhang): a complex system of meaning of a text or expression involving all the mental states of the author (goals, beliefs, past experiences) and their current environment (physical, social), and presumably those as anticipated in the reader or student*
>
> *Adapted from the Stanford Encyclopedia of Philosophy*

Purpose

The *CMF Nexus* design interface has been developed as a set of tools for the Cambridge Maths design team to use behind the scenes. However, this initial interface will be the root of our efforts to create tools for others.

The Vretta team is the first group outside the Cambridge Maths team to use this interface, and it would be especially helpful for us if you could keep a brief record of the experience you have with it. We're interested mainly in your ways of working from your perspective; less about interface details and more about what you need to do with the content and why.

How much to say

Please be brief, we don't want this to distract you from your main efforts. A short summary of whatever stands out after you spend a session using *CMF Nexus* would be good – but feel free to take notes as you go if you prefer. There are also some usability aspects we don't need to address at all because we're not to the point of refining a design for end-users yet. For example, we don't need to know about everything that is non-intuitive at first – that might be everything! – but some detail might be useful if something is problematic over repeated use, or if there's something important that can't be done yet.

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Specific questions from the Cambridge Maths writing team:

1. A little about the experience and role of the user: Are they currently a teacher, a task designer, an assessment specialist, etc.? Do they have some combination of roles in their background?
2. How do you choose tasks from your database to attach to something in ours?
    a. What part of the information in *CMF Nexus* tells you what you need to know in order to choose which of your tasks to attach and where?
    b. What information in your own database helps the most when linking to ours?
3. Where tasks should be attachable: do you find one or several of these to be most meaningful:
    a. A task linked to a specific student action in a waypoint
    b. A task linked to multiple student actions in a waypoint
    c. A task linked at the waypoint level, not specifying which student action(s)
    d. A task linked to a group of waypoints
4. Are there special considerations for linking activities, as groups of tasks, if you find that is relevant for you?
5. Do you find yourself attaching different types of tasks to particular types of waypoints (that is, landmark waypoints, exploratory waypoints, or waypoints which are not of a special type)?
6. What kinds of things are you linking to *CMF Nexus* (videos, explanations, other materials in addition to tasks or assessment items)?

How does this fit into our design process?

We are building narratives for different key uses of the Cambridge Mathematics Framework. This project will contribute to narratives for mapping assessment items and tasks to the CM Framework.

In addition to the narrative, we create models of how each of a user's sub-goals maps back to design features, design principles, and theoretical frameworks behind the design, and maps forwards to specific outcomes and impacts (once projects have been completed/implemented). We observe and/or record:

- Goals
  - Larger goals: what did the *CMF Nexus* user aim to achieve?
  - Smaller goals: As they go, what are the smaller steps they're taking to reach their larger goals?
  - Goals they successfully met
  - Goals they found they couldn't meet with the tools provided but which seem like they should be possible to meet given the underlying data available
  - Goals that were important but couldn't be met because of something lacking in the underlying data
- Summary of actions related to getting information the user needs:
  - looking for something
  - acting on something
  - abandoning an attempt
  - finding a workaround
  - doing frequently
  - finding the most useful
  - finding the most annoying

We link these goals and actions to our design principles and the theories of learning and design which inform them. A map like this, sometimes called a conjecture map or logic model, can be used by designers to evaluate and explain a design – as well as to refine design principles and even underlying theories – in a way that is generalisable enough to be potentially useful to other design efforts.