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Mathematics

Ontology

Structure and meaning in the Cambridge Mathematics Framework: a summary

Others in this series

- Building the research base: a summary
- Formative evaluation: a summary
- Research-informed design: a summary

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Summary

- The Cambridge Mathematics Framework aims to show internal consistency of meaning and of the mathematical ideas presented. This consistency has been refined and is recorded in our ontology.
- The Cambridge Mathematics Framework contains different levels of organisation. Layers within the CM Framework show how different Framework concepts are connected.
- The CM Framework layers include the Mathematical Ideas layer, Research layer, Glossary layer and other CM Framework layers. There are also add-on modules outside of the CM Framework which enable us to investigate additional uses of the CM Framework.
- The ontology has been informed by design methods in education. It follows the general cycle of ontology development, which includes building the general concept and refining it in an iterative way.
- When evaluating the CM Framework, we consider its purpose, potential usefulness, perspectives of the CM Framework designers and the relevant perspectives of those evaluating the ontology.
- Research Summaries show our application of the ontology, because they contain interactive portions of the CM Framework. Feedback from external reviewers may slightly alter the nodes in the research base and the contents of the live versions of Research Summaries.

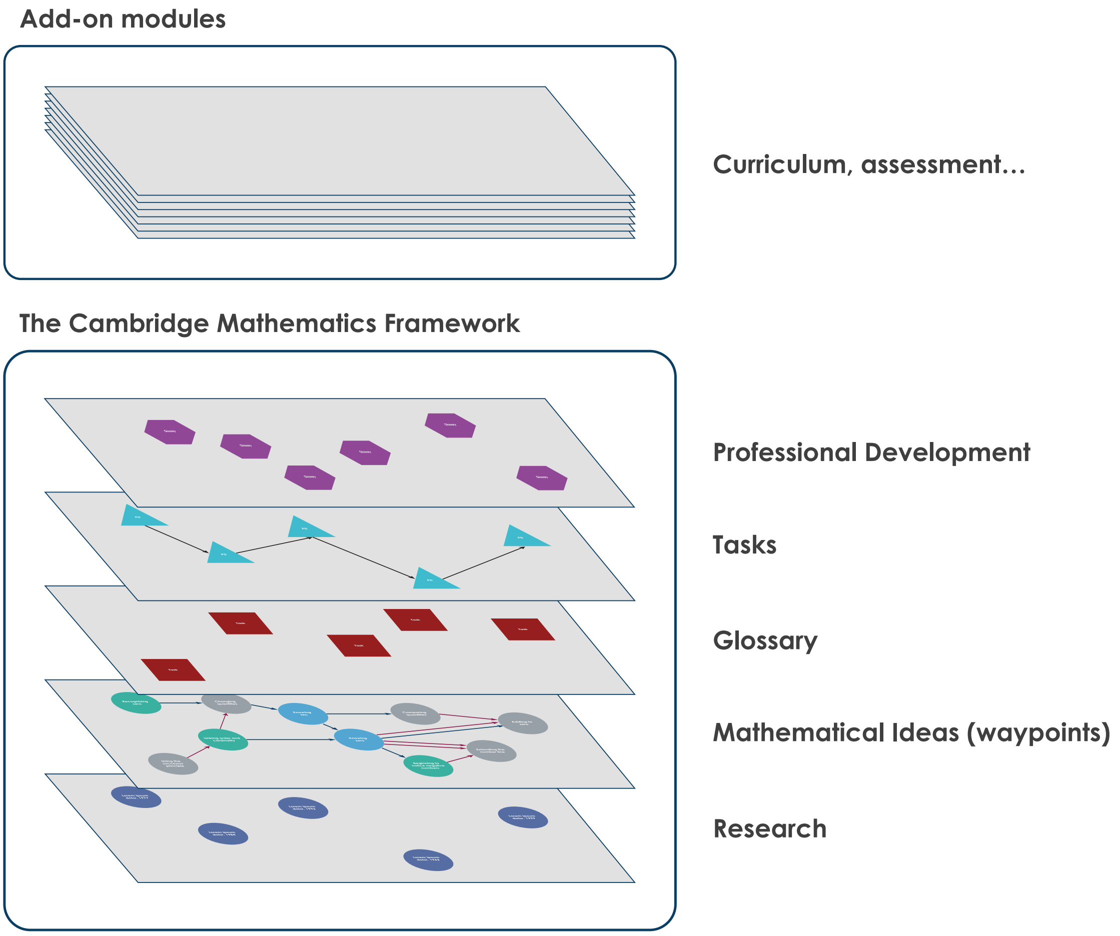
Introduction: ontology in the Cambridge Mathematics Framework

In order to support coherence in mathematics learning, the Cambridge Mathematics (CM) Framework should show internal consistency of meaning, which in turn should be consistent with the nature of the mathematical ideas being represented. This consistency has been refined as part of our design process and is recorded in our ontology. This document outlines the ontology used by the Cambridge Mathematics team in the structure of the database and in the design of the authoring tools underlying the CM Framework.

What is the structure of the CM Framework like?

The Cambridge Mathematics Framework treats mathematics as a web of ideas in which different meanings can be found at different levels of organisation. This web is built on a graph database (Neo4j), in which mathematical ideas are expressed as nodes (points) and edges (connections between the points). We have developed a web-based platform called *CMF Nexus* (Stevens et al., 2019), which allows us to search, filter and visualise the ideas expressed in the CM Framework, as well as view different levels and types of information as connected layers[1]. Figure 1 shows layers as a way of imagining how different CM Framework components can be linked together, and how other add-on modules outside of the CM Framework can be mapped to specific CM Framework components.

Figure 1: Different layers within the Cambridge Mathematics Framework and external add-on modules



[1] Described in Methodology: Research-informed design (Jameson, 2019b)

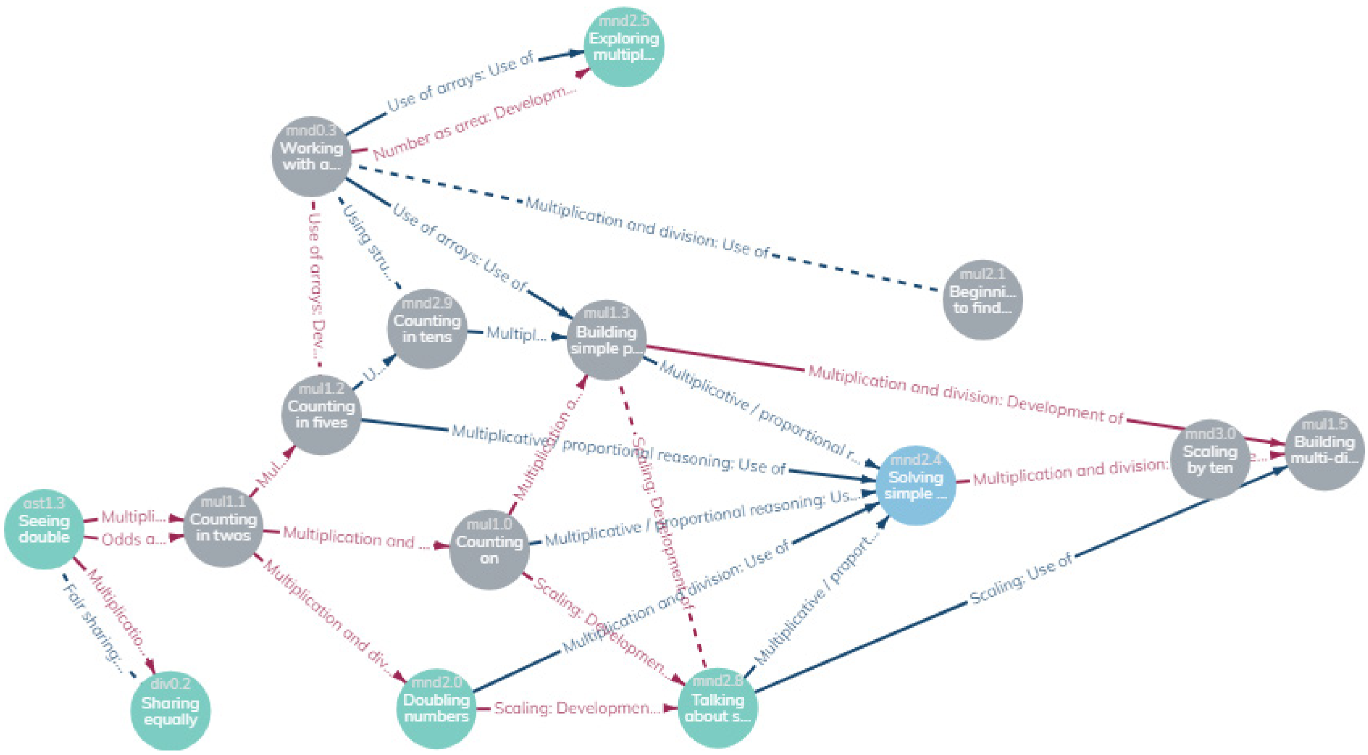
Other planned modules:

- Curriculum Statement layer,
- Landmarks layer,
- Subdomains layer,
- Etc.

Mathematical Ideas layer – waypoints

This CM Framework layer comprises mathematical ideas and relationships. The nodes in this layer are called waypoints. There are different types of waypoints: exploratory waypoints (shown in green in Figure 2) often come at the beginning of a theme and may suggest where ideas can be played with in a less formal way. They may introduce students to a concept and provide essential tools for understanding that concept. Landmark waypoints (shown in blue in Figure 2) are points where ideas are brought together and synthesised, therefore the whole experience may seem greater than the sum of its parts.

Figure 2: Example of a set of waypoints in the Mathematical Ideas layer



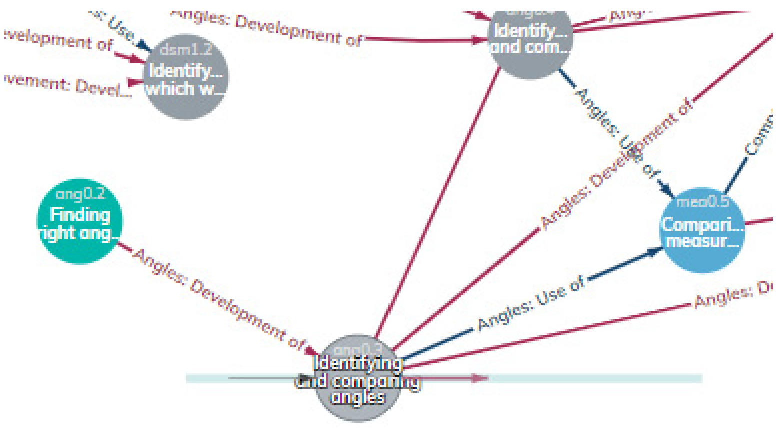
Individual waypoints may involve different mathematical concepts, processes etc. The structure aims to offer as much support as possible to its potential users in making decisions while remaining flexible, especially for those who may be planning the order in which they may teach an idea or deciding how long to spend on an idea.

Visualising and ordering waypoints

In Figure 2, the waypoints on the left lead into and contribute to those on the right. We have chosen to give the map a left-to-right ordering because we are representing the ways in which mathematical ideas contribute to one another. This is not definitive, as there are more waypoints in the CM Framework than a single curriculum could cover and there are often many different routes to a specific mathematical idea. The map does not represent a definite journey that all students would take, but aims to help designers, teachers and other CM Framework users to keep a conceptual picture of mathematical ideas in mind when planning their own routes and timings. It may be pedagogically useful for some to go back and forth between ideas and to skip some waypoints in some situations to better highlight others. CM Framework users should consider individual contexts, resources and the best ways to utilise the CM Framework.

Although we can adjust the order of waypoints manually, CMF Nexus will show waypoints in a provisional order from left-to-right by default. The writers can change waypoint positions within approximate ordering zones based on the lengths of the paths that lead into them. Figure 3 shows how the waypoint ‘Identifying and comparing angles’ can be placed anywhere along the left-to-right space indicated by the light-blue line.

Figure 3: Example of the placement bar feature: a light blue bar suggests a left-right ordering zone for the waypoint ‘Identifying and comparing angles’



Waypoints – how do they link to our design principles and to student actions?

Previous research has attempted to develop maps of learning and mathematics (e.g. Black, Wilson, & Yao, 2011; Daro, Mosher, & Corcoran, 2011; Maloney, Confrey, & Nguyen, 2014). For example, Michener (1978) developed a framework which mapped mathematics understanding. Although developed rigorously, it has so far been applied to limited areas of mathematics learning.

In order to describe other connections within the areas of learning, knowledge and mathematics, we give examples of student actions. These are derived from a task design framework which supports the building of conceptual understanding in mathematics (Swan, 2014, 2015). In the Cambridge Mathematics Framework, we intentionally focus on examples of actions that students could do rather than the specific things they produce. Our aim is to convey information to CM Framework users about *how* students might act whilst learning. Focusing on student actions instead of outcomes enables us to offer support for designers, teachers and other users of the CM Framework without implying which pathways students are more likely to follow as we do not have data to support such claims. Such flexibility recognises that students may engage with different ideas in different ways and at different times (see Table 1 for a description of student actions).

Table 1: Student actions in the Cambridge Mathematics Framework, adapted from Swan (2014, 2015)

Category	Student actions	Description
Procedural Fluency	Performing	Memorise and rehearse
Conceptual Understanding	Classifying	Sort, classify, define and deduce
	Representing	Describe, interpret and translate
	Analysing	Explore structure, variation, connections
	Arguing	Test, justify and prove conjectures
	Estimating	Use a sense of magnitude to make sensible predictions
Problem Solving	Modelling	Formulate models and problems
	Solving	Employ strategies to solve a problem
	Critiquing	Interpret and evaluate solutions and strategies

Mathematical Ideas layer

Waypoints are linked by edges, which represent a relationship between them. These relationships (themes) are identified as as either a) the development of, or b) the use of, a named mathematical theme.

Research layer (nodes and edges)

To develop waypoints and themes we use research evidence from literature reviews and collaborate with researchers and curriculum designers in areas where less research is available and/or implications may not be clear. Team members also consider their own experiences of classroom teaching and resource development to develop some ideas if no research is available.

Our research layer includes

- *Research Summaries* – which record the basis for the content and structure for a specific selection of nodes and edges (a saved search),
- *research nodes* – which correspond to specific research sources and contain metadata characterising them, and
- *research edges* – which connect the research nodes to their corresponding waypoints. They also describe if a source has been a main or a secondary influence on the waypoint or Research Summary it links to.

The Research layer contributes to our transparency principle (McClure, 2015). For more information about our research methods, please check Methodology: Building the research base (Jameson, 2019a)

Glossary layer (nodes and edges)

Key mathematical terms identified by the team during the writing of the CM Framework are defined in glossary nodes and linked to the waypoints in which they appear. This is a work in progress, informed by our CM Define It app.

Other CM Framework layers in development;

As shown in Figure 1, there are other planned CM Framework components, which include task design and professional development layers. We are carrying out small-scale collaborations which act as case studies to inform the design and development of these. We imagine that the task layer will contain some tasks to be linked to single nodes and others that may be linked to multiple nodes, reinforcing the interconnectedness of mathematics.

Add-on modules outside of the CM Framework

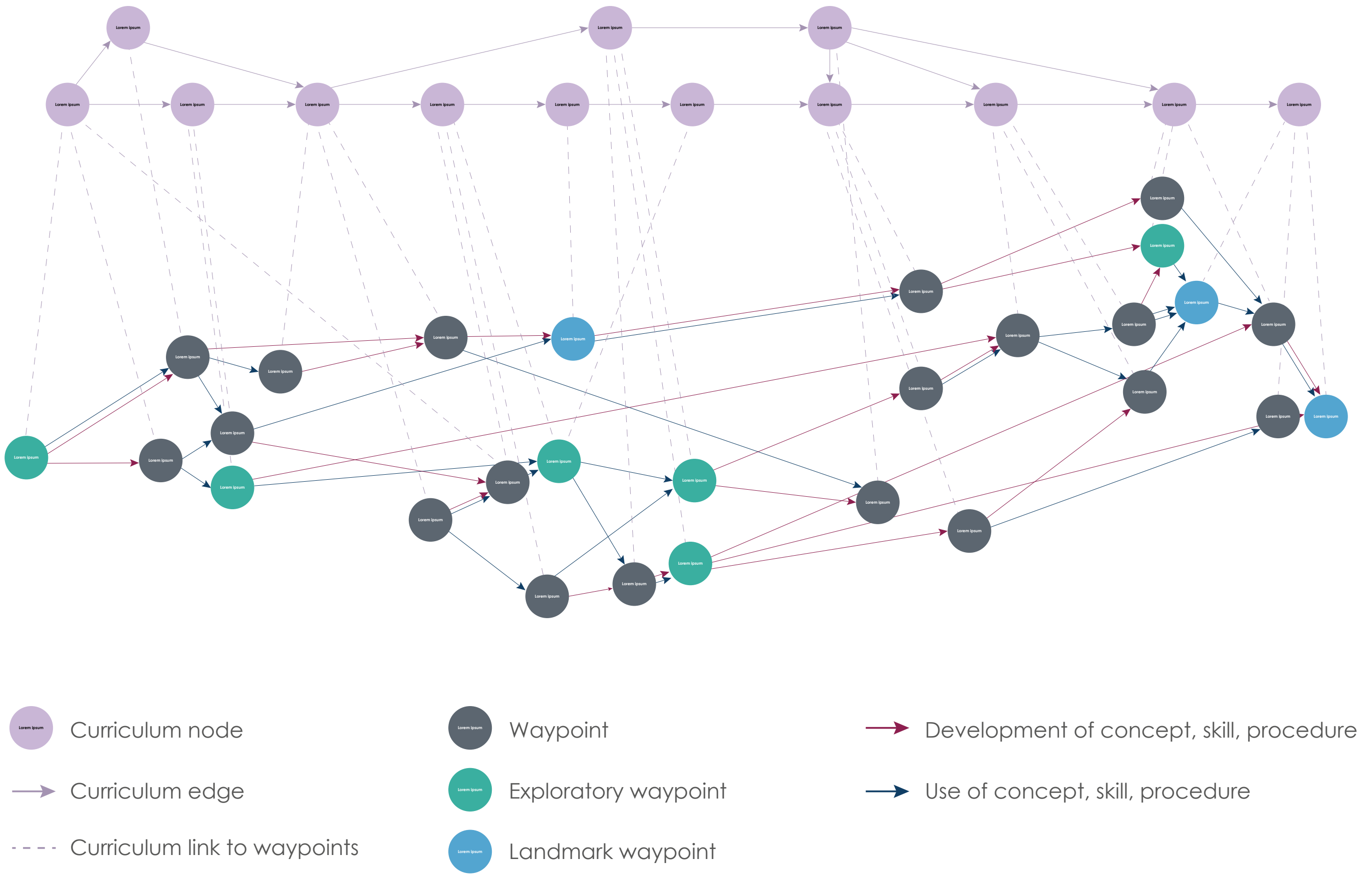
We are also building examples of other layers outside of the main CM Framework, linking back to the underlying waypoints and research base. In addition, we are experimenting with grouping waypoints into categories for different purposes, such as showing all landmark waypoints in their own layer. These groupings are not a main part of the CM Framework (unlike the Mathematical Ideas layer or the Research layer), but they do enable us to investigate additional uses of the CM Framework.

Curriculum layer

Through case study work we create Curriculum layers, which allow us and our collaborators to explore other uses of the CM Framework, such as curriculum mapping or design.

Figure 4 on next page

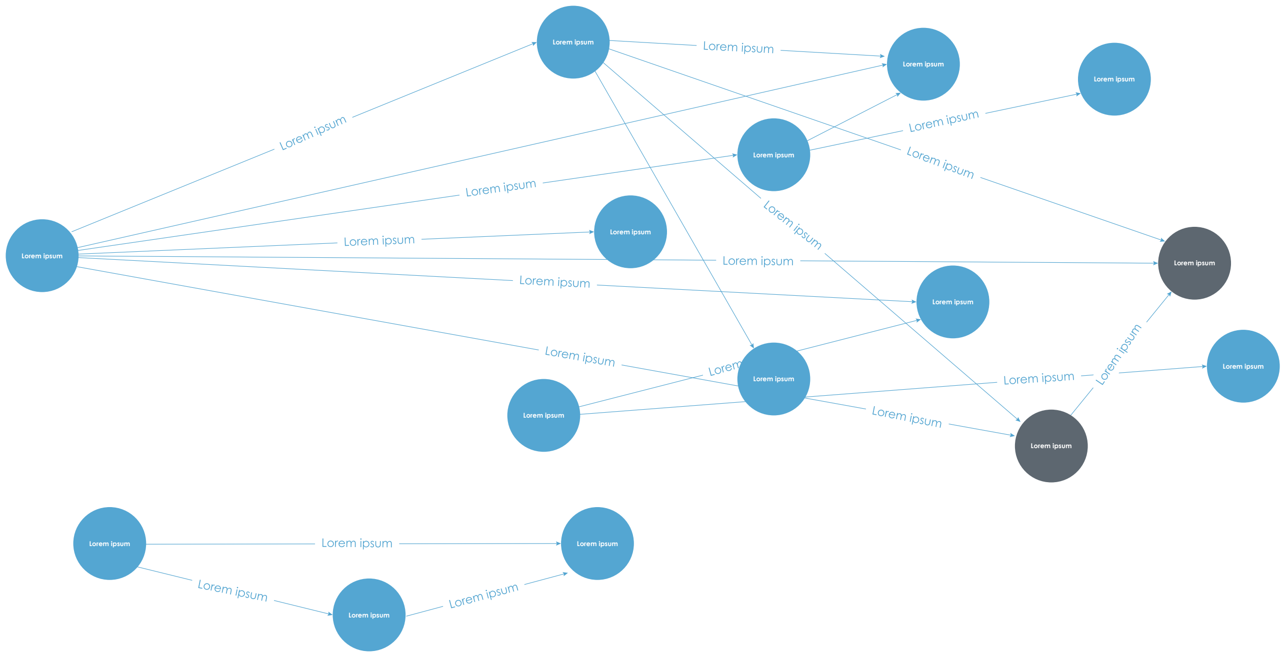
Figure 4: An example of the Curriculum layer, showing curriculum statement nodes and edges (pink, top) and waypoints and theme edges (all non-pink, below)



Landmarks layer

Zooming out from the detail focuses on the *landmarks*. *Landmark edges* demonstrate the shortest path of waypoints connecting two landmarks, and each path lists the collection of themes it contains.

Figure 5: A selection of landmarks in the Landmarks layer



How was the ontology developed and how is it used?

Our ontology draws on design methods in education. Specific aspects of design research that are appropriate to our project include: linking specific design priorities and choices to theory, using initial design work to develop design principles that inform ongoing work, iterative cycles of design in which feedback is incorporated into new versions, and participation in design by experts in multiple relevant communities (Barab & Squire, 2004; McKenney & Reeves, 2012; van den Akker, Gravemeijer, McKenney, & Nieveen, 2006).

Our ontology design followed the general cycle of ontology development described in Fürst, Leclère and Trichet (2003, p. 80). This means that we:

- built an informal conception of mathematics learning and relationships we wanted to model, and then
- built an initial conceptual map and revised it many times; gradually agreeing on the structure of mathematical ideas.

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How do we refine our ontology?

The Cambridge Mathematics team aims to interpret and express ideas consistently so that they can be understood and used by others. The CM Framework viewing and authoring tools we have developed in CMF Nexus allow us to refine the ontology when needed; for instance, when we want to express something important which does not yet 'fit' across the structure. This has enabled us to experiment with new categories and ideas, some of which have been incorporated into our ontology.

How do we evaluate our ontology?

We considered several factors:

- the purpose of the CM Framework;
- its potential usefulness;
- the perspective of the CM Framework designers; and
- the relevant perspectives of those evaluating the ontology (Barlas & Carpenter, 1990).

We conducted a Delphi study[2] so that external professionals with experience in national-level mathematics curriculum research and design could evaluate the trustworthiness of the ontology and its potential value. We continue to do this through ongoing case studies.

How do we use the ontology to write the CM Framework?

Framework writers review the literature appropriate to each area of the CM Framework[3] and use the ontology to make decisions about what is important to include, which waypoints to express individually and which to express as a combination of waypoints. For Research Summaries that cover specific areas, the team notes consensus and lack of agreement between sources. The influence of research sources on our work is evaluated through the external review process[2]. Connections between areas within the CM Framework are made during initial synthesis, discussion between writing team members and sometimes through external review. We have also developed a set of web-based tools in CMF Nexus which allows us to visualise, create and edit text and structure in the database.

[2] Described in Methodology: Formative evaluation (Jameson, 2019c)
[3] As described in Methodology: Building the research base (Jameson, 2019a)

How do we use the ontology to support a shared frame of reference?

The ontology is an artifact; designing it with our potential users in mind ensures that we make the work accessible, and helps us to recognise when a new feature is needed to express an important element of the CM Framework.

Our approach has been influenced by: models of pedagogical content knowledge (Shulman & Shulman, 2004), models of co-design practices for knowledge representation (Sanders, 2002), individual and social processes of technology-mediated knowledge building (Stahl, 2006) and theories of boundary objects (Star & Griesemer, 1989) and boundary negotiating artifacts (Lee, 2005).

Example of how the CM Framework ontology can be applied

Before the CM Framework is available to external users, Research Summaries[4] show our application of the ontology, because they contain interactive portions of the CM Framework (saved searches). To see an example of a Research Summary that has been reviewed internally and informally by external evaluators, please see Ontology: Structure and meaning in the Cambridge Mathematics Framework (Jameson, 2019). The contents of the live versions of Research Summaries may change depending on feedback from external evaluators and on any other changes made to the nodes in the research base.

[4] Described in more detail in Ontology: Structure and meaning in the Cambridge Mathematics Framework (Jameson, 2019)

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