CAMBRIDGE Mathematics

Case study micro-report

Writing a textbook chapter using the Cambridge Mathematics Framework

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Case study micro-report: Writing a textbook chapter using the Cambridge Mathematics Framework

Sections in this document

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We describe the use of the Cambridge Mathematics (CM) Framework in writing a mathematics textbook chapter which needed to be aligned with an unfamiliar curriculum. The writer diarised her experiences and presented us with a narrative describing the writing process. We developed a list of themes and key user actions based on this narrative, which we linked to our design principles and underlying theoretical assumptions. This list has helped us to identify the basic affordances and support we will need to provide to textbook writers so that they can use the CM Framework effectively for this purpose.

Introduction

This paper is part of a series of case study reports and the background for the series as a whole is provided in a separate paper[1]. In this report, we present a textbook chapter-writing case study. We consider textbook writing to be an important resource design context for the use of the Cambridge Mathematics Framework. To support students having coherent mathematical experiences, we want to enable the CM Framework to serve effectively as a common frame of reference across design contexts in mathematics education – curriculum design, resource design and teaching[2]. This case has helped us to learn what processes are involved when using the Framework in this context, and what support may be useful to provide so that these processes can be made as straightforward and streamlined as possible for all users of the CM Framework.

Case context and goals: Writing a textbook chapter aligned with a given curriculum

This case features the process of writing a Primary grade 4 (~ age 9) textbook chapter on the topic "Calculating the area of a rectangle" in such a way that it would provide students and teachers with a coherent mathematical narrative, aligned with the curriculum the textbook is intended to support. This work was done by a member of the CM Framework writing team as part of a pilot project collaboration between a non-European jurisdiction, Cambridge University Press (CUP) and Cambridge Mathematics. This jurisdiction's curriculum was sufficiently different from curricula in the UK that the writer could not rely on preconceptions from her teaching experience in the UK. The CM Framework would therefore be useful as a reference and design tool, helping her to bring her experience as a subject specialist to bear in a new context.

[1] Background for the case study micro-report series (Jameson, 2019a)
[2] Shared perspectives on research in curriculum reform: Designing the Cambridge Mathematics Framework (Jameson, McClure, & Gould, 2018)

The writer identified a set of design questions related to the development of support for resource design uses:

1. How can users find what they need to write a textbook chapter?
2. What can help users prioritise their attention when reviewing content they find?
3. How can users work with the content from multiple perspectives?
4. How can users know what is meant by important vocabulary terms in the content?
5. How can users focus their initial map down to a final map?
6. How does the final map help them to write a textbook chapter?

In addition to the design questions specific to this case, we would expect the CM Framework, as a knowledge map, to support users in more general ways (Eppler, 2004). We can examine the use of the CM Framework for textbook chapter-writing in light of the typical affordances of knowledge maps (see Table 1) to see whether we are achieving some of the benefits we might expect.

Table 1: Expected affordances of knowledge maps; all are relevant to some degree in this case

	Combined user groups	This case
Help designers to communicate ideas about knowledge to others; make tacit ideas explicit and present ideas in a form that users can relate to (Eppler, 2004; Vail, 1999)	✓	✓
Help users to "remember, comprehend, and relate knowledge domains through insightful visualization and aggregation of information" (Eppler, 2004, p. 200)	✓	✓
"[M]ake information actionable in new contexts, connect it with previous experiences" (Eppler, 2004, p. 189) – that is, professional learning and transfer	✓	✓
Help users to evaluate what knowledge is available for decision-making, and from what sources (Eppler, 2004)	✓	✓
Help users to see concepts within a bigger picture and to switch between multiple perspectives (Eppler, 2004)	✓	✓
Help users to evaluate and compare sets within knowledge domains – examining what knowledge is available, from what sources, and with what justification (Eppler, 2004)	✓	✓
Provide a "common framework" when searching for or contributing "relevant knowledge" (Eppler, 2004, p. 190), which itself supports professional learning	✓	✓
Contribute to the field by providing a big-picture perspective and a research base with respect to ideas that people in different roles may hold in common	✓	✓

	Combined user groups	This case
Relate the big-picture perspective to different levels of underlying detail (Eppler, 2004)	✓	✓
Support professional learning in practical contexts: "just-in-time" (Vail, 1999, p. 23)	✓	✓

Role of this study in our evaluation plan

This case was the first in which we developed a set of processes for a key use of the CM Framework and documented them from start to finish. It helped us to develop ways of conducting, observing and reporting such cases and feeding results back into our design process, which we have applied in more recent case studies. We were afforded the opportunity to see how this task might be accomplished using our existing design tools and what additional support might be necessary to streamline or simplify the process.

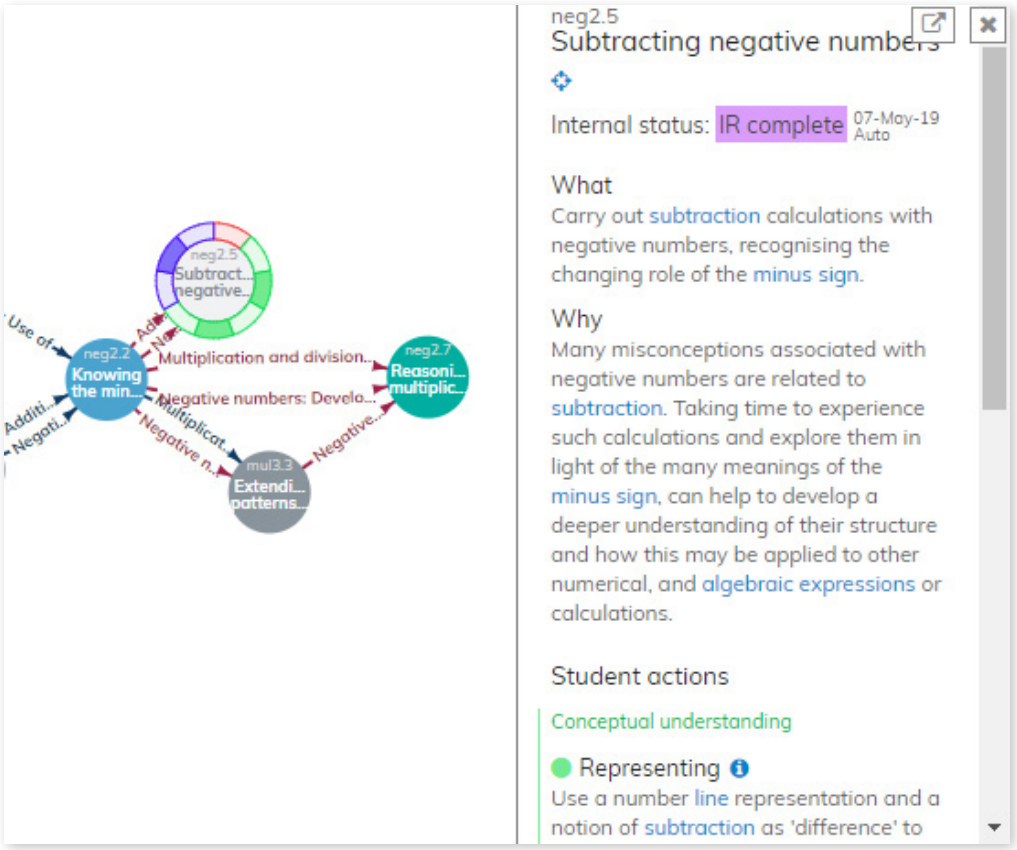
Methods and materials

Cambridge Mathematics structure and content[3]

The CM Framework team worked with the network of mathematical experiences in the CM Framework. This network is our interpretation of the knowledge of students' learning which has been developed in the mathematics education community through empirical research and practice. The content used in this case study came from this network. Mathematical ideas and key relationships between them are represented in a network of *waypoints*, which we define as "places where learners acquire knowledge, familiarity or expertise" (Jameson et al., 2019, p. 4). Waypoints have titles, descriptions and *student actions*, which are examples of the kinds of things students might do to help them build an understanding of the content at a waypoint. An example of a waypoint and its content is shown in Figure 1. We call the relationships between waypoints *themes*, and in the CM Framework they represent either the development of some part of an idea from one waypoint to the next (with an arrow pointing from left to right in the direction of development), or the use of one idea contributing to understanding of another (an undirected relationship)[4].

[3] This subsection is reproduced from *Using the Cambridge Mathematics Framework to refine the UNICEF-Cambridge Curriculum Progression Framework (Mathematics)* (Jameson and Horsman, 2019)
[4] Waypoints, themes, and types of student actions are described in *Ontology: Structure and meaning in the Cambridge Mathematics Framework* (Jameson et al., 2019)

Figure 1: Example waypoint with content ('what,' 'why' and 'student actions)



Research Summaries are documents in which a subset of this network of waypoints and relationships is embedded and shown as a map; they "tell the story" of the content and structure of a set of waypoints and themes. Each Research Summary includes three potential elements of interest for this case:

1. a summary of the landscape of research literature informing the content,
2. the embedded CM Framework content and
3. a description of how our interpretation of the sources in our research base has led to the structure shown in that Research Summary.

We determined that Research Summaries would be the most accessible way, at our current stage of interface development, to present CM Framework content for external use. They can be viewed as static documents or through the online CM Framework platform, CMF Nexus (Stevens et al., 2019). In CMF Nexus, the embedded content can be viewed dynamically so that the content for each waypoint can be viewed in full one waypoint at a time, or it can be output as a spreadsheet for users to scan across waypoint content in a different way.

User backgrounds

In this case the writer was the sole user. She had authored the CM Framework content she was working with to write the textbook chapter, but had not used it to build a mathematical narrative apart from the topic-area Research Summaries she had written. She had previously written textbooks and developed other forms of pedagogical resources for teachers, and facilitated professional development sessions nationally and internationally. She had also taught in lower and upper secondary mathematics classrooms and advised in primary classrooms for over 13 years, and was a department head for much of that time.

The pilot case protocol

The writer diarised her thoughts and actions, and consolidated these in a video narrative re-creating the process as a whole. This was presented as part of the Cambridge Mathematics Symposium in 2018. Building on her own categorisation of her actions in her narrative, we completed one round of open coding of her video narrative to pull out user intentions, actions and other themes, and outlined all themes in sequence. The sequence and the list of key user intentions and actions was fed back to the design team.

Time frame

The activities described here occurred over six weeks in 2018. This took place before the Vretta task mapping and *Learning Passport* curriculum design cases[5]. Whilst less of the CM Framework had been written at that time, the textbook chapter covered topic areas which were ready for use. The writing of the textbook chapter itself took approximately 20 hours.

[5] *Mapping tasks to the Cambridge Mathematics Framework with Vretta* (Jameson, 2019) and *Using the Cambridge Mathematics Framework to refine the UNICEF-Cambridge Curriculum Progression Framework(Mathematics)* (Jameson and Horsman, 2019)

Results

Narrative

Summary of the video narrative

The writer narrated a video in which she reproduced and described the processes she went through to develop the chapter. This is a summary of that narrative :

The writer started with an endpoint which was the topic of the chapter: "Calculating the area of a rectangle." She used it to build a big picture view, looking at the important points (landmarks) that led up to it. This view wasn't a guide to writing the chapter, but an overview to help her start putting that guide together.

She then looked in more detail, using CMF Nexus to automatically trace backwards along paths of waypoints which were connected to the endpoint, all the way back to foundational ideas. Then, working forwards along the paths from these foundational ideas, she examined the connecting themes and noted which seemed most important. She prioritised the content she found, removing lower-priority considerations from the map.

This process resulted in a map that was more tailored to the chapter's curriculum aims, was coherent and preserved the story of how students and teachers could reach the goal. Checking by working backwards resulted in further pruning. The final result was a map of content which she used as a guide for writing the chapter.

Video of the full narrative

A video of the full narrative is available on our website[6]. It includes an accompanying video screen capture from CM Nexus illustrating the use of the CM Framework (as it was in March 2018) for each step described in the narrative.

Selected themes from the narrative

Table 2 presents the main themes which emerged from coding the video narrative. These fall into three categories: the problems faced by resource designers when beginning to use the CM Framework, the goals which might frame how a writer starts using the CM Framework, and the writer's intentions and actions during the resource design process.

[6] The story so far (video under 'The story so far;' the writer's narrative starts at 12:47)

Table 2: Selected themes from the writer's narrative

1. Problems that resource designers face
1. Finding a way to develop a resource using the CM Framework
2. Too much information in a full map – how to focus on a reduced amount
3. How to get more than one overview perspective on the map
4. How to prioritise information in the map
5. Knowing what to do with the waypoints in the reduced map
6. Speaking the same language – defining vocabulary terms
2. Goals for use (broader intentions of the resource designer)
1. Confirmation exercise – already had a picture in mind; or,
2. Starting exercise – building a picture of chapter without a pre-existing one
3. Resource design process – full list of user intentions and actions in narrative order
1. Identify curriculum context for learning objective
2. Read existing curriculum
3. Find a waypoint in the CM Framework corresponding to the end goal of the chapter
1. Become familiar with contents of the CM Framework
1. Follow a particular theme
2. Keyword search
3. Read the research summary attached to it
4. Understand how the CM Framework characterises that end goal
5. Big picture view of endpoint
1. Work backwards from endpoint at landmark level
1. Identify landmarks
2. See connections to other areas of mathematics
3. See strength of link – number of steps between landmarks
4. See which themes connect the landmarks
2. Examine the connecting themes at the landmark level more closely
1. Zoom in on a link between landmarks to see waypoints and themes involved

4. Resource design process – full list of user intentions and actions in narrative order

6. Examine the map in more detail
    1. Look at what the endpoint leads to later on
    2. Look backwards at everything that came before the endpoint
        1. Investigate interesting characteristics of the map of waypoints
        2. Identify and investigate key connections
        3. Identify and investigate key waypoints
7. Examine the map in more detail
    1. Check to confirm that expected things are present
    2. Look for characteristics of connections that jump out
        1. Look at what connections are frequent in the map
    3. Isolate characteristics of interest for a closer look
    4. Understand why key features are present in the map
    5. Decide what to prioritise
    6. Filter out what is not needed for this area of focus
    7. Produce a map that is more focused but is still coherent and includes the right details
8. Work backwards step by step
    1. Expand one step back
    2. Look deeply at a waypoint and its role in the themes it is connected to
    3. Refer to linked data, e.g. in the glossary
9. Writing a chapter from a focused, coherent subset map of the framework
    1. Find the starting point in the sub-map using exploratory waypoints
    2. Work forwards from the starting point and make decisions at key points along the way
10. After writing, use the subset map and Research Summaries to provide transparency in decision-making; explain the structure and content of the chapter and what influenced it

Based on these themes, the major user actions which emerged in this case were

- determining an end goal,
- frame-switching[7] (zooming in and out to look at details and the big picture)
- seeing important connections,
- following connections backwards and forwards, and
- creating a focused, coherent subset map

Discussion

This was an early case, and the textbook chapter writer was working with CM Framework content which she had authored. The most important overall questions at the time were whether it would be possible to use what had been written in the CM Framework so far to produce a textbook chapter, and how it could be done. Addressing these questions has helped us judge the feasibility of using the CM Framework for this purpose and has provided a starting point for developing further support for users engaged in textbook writing.

This case also contributes to our broader process of prioritising refinement and support for features of the CM Framework itself and the tools and interfaces in CMF Nexus. By comparing this case to others, the Cambridge Mathematics team can use the sequence of user intentions and actions in Table 2 to help determine which features of the CM Framework are required across different uses and which features might be needed for particular but essential uses. For example, frame-switching was also later identified as a key user action in the Vretta task-mapping case, and expanding and reducing the map view were key user actions in the *Learning Passport* case study.

Many of the expected benefits of knowledge maps were realised in this case, though not all completely applied (see Table 1; benefits which did not apply strongly in this case are unchecked). With only a single writer involved, the CM Framework was not used directly as a communication tool between curriculum or resource designers, as it was later on in the *Learning Passport* case study[8]. However, it did facilitate communication of ideas indirectly between the designers of the CM Framework (and their sources and collaborators), the textbook author, and the teacher and student audiences of the textbook. The CM Framework helped the writer to apply teaching experience to writing for a curriculum context which was new to her, with which she needed to align her work, quickly and appropriately. The CM Framework

[7] Also reported in the Vretta task-mapping case study
[8] Reported in the Learning Passport curriculum design case study

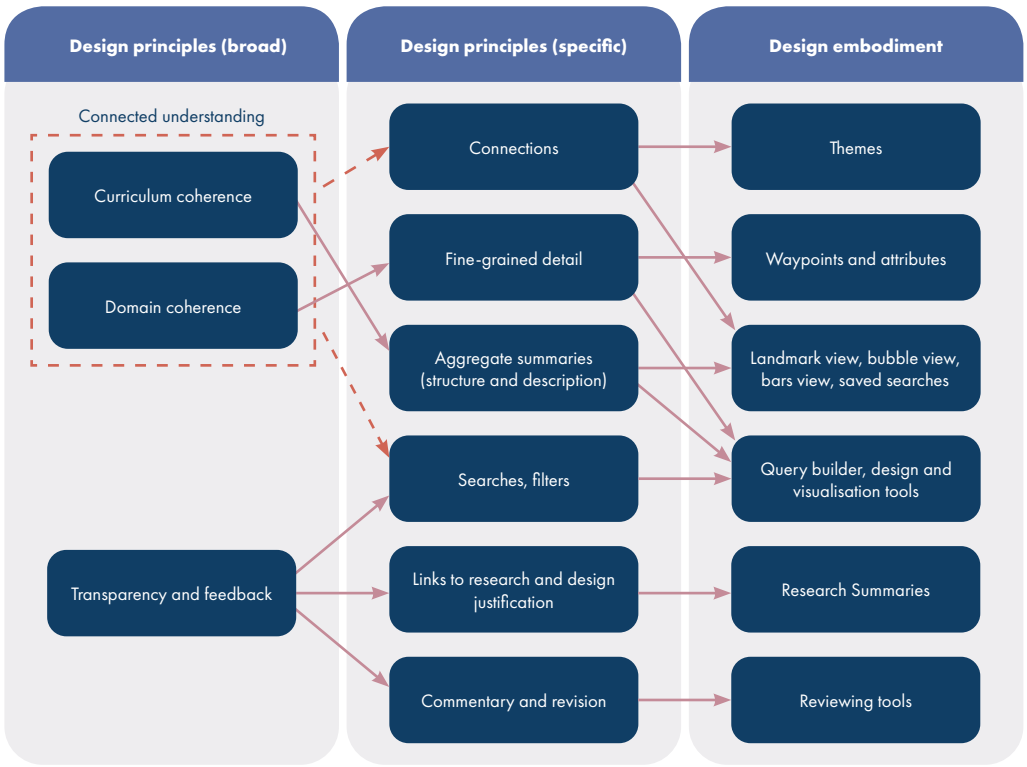
helped the writer to apply her extensive teaching experience quickly and appropriately to the new experience of writing a textbook chapter for an unfamiliar curriculum.

Five major actions emerged from the writer's narrative (see Fig. 2). These depended most on the features of tools in CMF Nexus which enabled

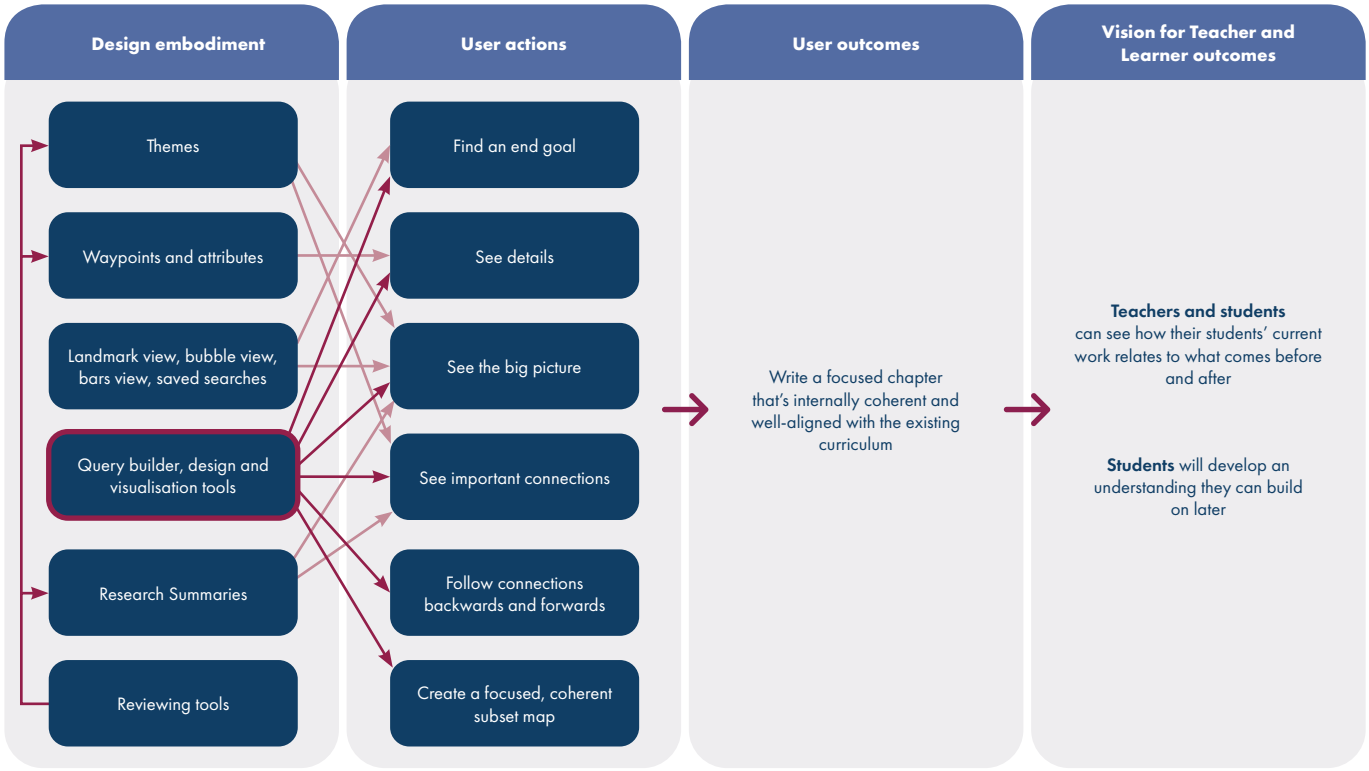
- searching for specific content by key word and by expanding backwards or forwards from content in the map view,
- manipulating the map visualisation to add or remove content, and
- switching views between content details, the story of those details in Research Summary text and the larger picture.

They also depended on the features of the CM Framework structure and content[9] which allowed a coherent "big picture" to be formed flexibly from a larger pool of available content, and which made the presence and nature of connections between mathematical ideas explicit.

Figure 2: Case-specific logic model of CM Framework design focusing on major user actions for textbook chapter-writing



[9] *Shared perspectives on research in curriculum reform: Designing the Cambridge Mathematics Framework* (Jameson, McClure, & Gould, 2018)



Logic models like the one informed by this case (Fig. 2) are a useful way for us to understand what our design features enable users to do. This helps us to determine whether and how the intended affordances of the CM Framework that we have envisioned as its designers are being realised in practice. We can use them to explicitly keep track of the links between the actions of CM Framework users as they work towards their goals, the features of the design which enable those actions ("Design embodiment" in Fig. 2) and the design principles informing those features. Knowing the links between our assumptions and outcomes in this way can inform any changes we may need to make in cases where expected design outcomes may not be realised. It can also help inform the work of others who may be facing similar design challenges or developing similar design principles[10].

[10] See 'Developing the rationale for specific design choices' in *Methodology: Research-informed design* (Jameson, 2019b)

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