

AN UPDATE ON THE CAMBRIDGE MATHEMATICS FRAMEWORK



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PREFACE

When the Cambridge Mathematics initiative was first launched in March 2015, two documents were produced for discussion. The first was the *Manifesto* which stated in general terms what we were intending to do. Apart from a change in tense, that document still stands.

The second document was entitled *Cambridge Mathematics Framework* and, as well as including elements from the *Manifesto*, set out in some detail our initial ideas of ways in which the Framework might be designed and enacted. Some two and a half years later, and halfway through the initial five-year period, many of our original broad fundamental ideas still hold. During that time, we have been challenged and supported in equal measure. We have been fortunate to have met and engaged with colleagues who have been generous with their time and expertise and who have advanced our thinking in a way that we could not have achieved independently. We feel now is the time to share the progress we have made so far, the questions which are still intriguing us, and to invite still further challenge, support and discussion on our continuing work.

THE FRAMEWORK AND THE AIMS OF CAMBRIDGE MATHEMATICS

Ultimately, and as initially described, we intend that the Cambridge Mathematics Framework will connect the four key elements of mathematics education: curriculum, resources, professional development and assessment. Our work to date has focused on the Framework as a design tool that links these elements, and it is the work we have done on this that we would like to document here. We have built the beginnings of a multi-dimensional, connected structure influenced by theoretical perspectives, international evidence and empirical research, whilst keeping an eye on what new mathematics could be incorporated into a framework fit for the 21st century.

These aims of Cambridge Mathematics fit broadly within those of the wider University through contributing to education, and thus society. Together with our University partners, we are producing a framework which supports access for all learners: from those for whom the study of mathematics is an all-consuming passion to those who find learning mathematics highly challenging and largely irrelevant. Through scholarship, collaboration and consultation, we are bringing together a large body of research and expertise to inform our design and the decision-making processes underpinning it.

Framework design overview

The Cambridge Mathematics Framework is designed to be a common frame of reference for learning mathematics. Its purpose is to inform the work of the professional communities designing and enacting mathematics curricula. In particular, it supports designers and teachers to make use of the connected nature of the mathematics learning domain. When released, it will comprise:

- a guiding structure that determines what and how content is expressed in the Framework
- a database of mathematical content, defined, referenced, and exemplified as actions, informed by research synthesis and consultation
- an interface providing a set of tools for searching and visualising mathematical content and the research base
- connections to specific classroom activities, assessments and professional development resources.

One use of the Framework is as a tool for designing curricula and associated content. It contains multiple paths through more mathematics than could be covered in a single curriculum, and so a curriculum designer, aided by information from the research base, will make choices about which pathways to construct. Developers of teaching and learning materials will then interpret the pathways to design learner journeys that are optimised, and allow for maximum flexibility.

The guiding structure and content of the Framework are described in more detail below. Based on our starting context, the three most important principles guiding our design decisions are:

- **connectivity:** Making important connections explicit in a consistent way will help these connections to be referenced more easily, including those which may span multiple areas or otherwise tend to escape attention in existing curricula.
- **early experiences:** identifying activities that offer early exposure to mathematical ideas or practices
- **transparency:** In order to be able to make considered decisions, users should be able to know what and how the evidence base of literature and expertise has influenced any part of the Framework.

Design context and background

Our aim is that the Framework will provide a common or shared representation so that different stakeholder groups (for example, curriculum designers, textbook writers, professional development providers, teachers) find it easier to transfer knowledge among one other. Many fields recognise and engage with this class of problem, including languages, organisational studies, geography, political

science, economics, and anthropology. Shared knowledge representation has been shown to facilitate working between groups who have differences in their constraints and priorities, to make the most of their shared outcomes (DiSalvo & DiSalvo, 2014; Lee, 2005; Robutti et al., 2016; Star & Griesemer, 1989).

Shared models, like any model, are always likely to be more closely aligned to some parts of a real system than others because of the compromises involved in creating a simplified system. This means a model may alleviate some problems involving shared understanding while failing to address others. Therefore, it is important for us to keep in mind and communicate clearly the beliefs that shape our overall design approach. These are summarised in the following table.

Beliefs	Problem	Perspective	Design approach
Equity in education	Students who have limited or no access to teachers or resources need a clear and coherent curriculum to support independent working	Coherence can be improved and given a stronger grounding in evidence	Focus on increasing support for awareness of connectedness of mathematical ideas
Equity in education	Teachers with less content knowledge are often placed with lower-performing students, perpetuating the cycle	Teacher content knowledge may be enhanced by access to a map of mathematical experiences and their connections	Include tools, interfaces, and structural anchors that make the Framework content directly searchable and useful for teachers
Connected understanding	Different stakeholder groups acknowledge and privilege the underlying structure of mathematics to varying degrees	Holding understanding in common requires shared access to a common reference and contributes to improved design and teaching of curriculum and resources	Express connected content in a way that can be recognisable, relevant, and useful across professions with extra detail specific to each
Connected understanding	Adherence to canonical examples of particular mathematical ideas or structures may close down more appropriate options	Linking disparate content which has common mathematical structure can provide more options for decision-making in curriculum and resource design	Identify and link fundamental mathematical ideas, structures, practices, and ways of thinking across the Framework
Coherence	Lack of alignment and communication between stakeholder groups reduces the coherence between intended and enacted curricula, affecting learning	Users in different roles can make decisions about content defined in dimensions they hold in common, with extra detail for sense-making within each group	Incorporate face validation of content and structure into the iterative design process to evaluate usefulness for stakeholders
Coherence	Experiences may be introduced to students in an order that does not provide the best support for learning or refining key ideas	Showing dependencies can help users evaluate compromises according to needs	Be able to search and display options for sequencing or resource design, based on localised contexts, needs, constraints

Many other relevant beliefs, perspectives, and approaches on curricula exist in mathematics education communities, but those above, in particular, have informed our overall design (see table below). There are additional beliefs that strongly motivate our work (discussed previously in the *Manifesto*), including for example the value of mathematical thought in human experience for its own sake, as well as its instrumental roles in employment, citizenship, and creativity in other fields.

As with any such project (Confrey & Lachance, 2000), the design of the Cambridge Mathematics Framework is the result of priorities and constraints that arise from these particular beliefs, as well as the nature and limits of our backgrounds and past experiences. For example, our prior experience with mathematics curriculum design is largely Eurocentric, and our review of the literature is mostly limited to English-language publications except in cases where we communicate with international experts. We continue to expand our perspective beyond our own limits through formal and informal communication with members of stakeholder communities of practice, both nationally and internationally.

Users and uses of the Framework

In order for the Framework to have an impact on the issues identified above, we are designing it for main users in three broad categories, each with a profile of use according to the time frame or scope of the tasks they are undertaking (see table below). We expect that many users will be members of more than one category and might use the Framework in different ways depending on their current role.

There are additional categories of users who might nevertheless use the tools designed to support the three main categories, for example:

- **researchers:** for comparing curricula, characterising gaps in the literature, and identifying critical areas for funded work
- **assessment developers:** for evaluating the mapping between curriculum and assessment content
- **teacher educators and professional development instructors:** to identify content to investigate through a particular lens
- **students:** to help form goals, gain a perspective on past work and look ahead to future topics

User category	Time frame	Depth and breadth of use
Curriculum developer	Long: curriculum revision may occur in 5–10 year cycles	Breadth: may deal with aggregated information in the more detailed levels
Resource/textbook/ scheme of work developer	Medium: a few months to a few years depending on the resource	Breadth constrained to a portion of the curriculum; more detail but not the most detailed levels
Teacher	Short: a few days to a few weeks	Depth of content knowledge in targeted areas (but with occasional reference to horizon content knowledge)

The design tool that we are using to write the Framework is not appropriate as a tool for intended users. However, the underlying database together with our growing expertise in the affordances and constraints of the tool will inform the development (planned for the next two years) of user interfaces.

Design process: consistency and the use of research and feedback

The design of the Framework is informed by research and influenced by feedback in ways similar to those described for design research methods in education (McKenney & Reeves, 2013; van den Akker, Gravemeijer, McKenney, & Nieveen, 2006).

Research base

The Framework content is based on literature review and consultation with researchers and colleagues. Whilst, as a prerequisite for design, a systematic literature review in every area of school mathematics is not possible for us, many projects have undertaken portions of such a review and we refer to their work whenever possible. To identify important themes and findings, we are following a semi-structured review process that includes keyword database searches, purposive sampling according to syntheses, meta-analyses, recommendations from consultation, and breadcrumb searches starting from widely accepted texts such as recent research handbooks (Thomas & Harden, 2008). To facilitate expert review and our goal of transparency, we enter our sources, linked to the corresponding content, into the Framework database and we also record various categories of metadata. This makes it possible for writers, reviewers, and users to summarise and examine the influences that have contributed to specific areas of the framework.

Consistency and meaning

We have developed a guiding structure for positioning content in the Framework that allows us to make ideas explicit, set scope and boundaries, and find patterns. It is this structure that lays the groundwork for determining whether and how shared meaning can be conveyed between designers and users of the framework. In this way, it acts as an ontology (Schneider et al., 2011), which Gruber (1993, p. 199) defines as “the objects, concepts, and other entities that are presumed to exist in some area of interest and the relationships that hold among them.”

This ontology is not fixed but is something we are continuing to add to and refine. It is fundamental to the design of the Framework and so it is a key focus in our internal and expert review process.

External validation, value, and trustworthiness

For the Framework to be coherent it should express content across the breadth of the curriculum and with enough depth to be useful for reference by intended users. This should be in such a way that it can be agreed to be a valid representation of mathematics learning – concepts, processes, ideas, actions, etc. Our design process for initial content creation is meant to lay as much of a foundation for this

as possible, but we need to validate this before the Framework is used in schools. This raises the problem of evaluating a design whilst in process. Wenger et al. (2011) suggest that a combination of informal feedback together with formal indicators could provide complementary perspectives.

Currently we are evaluating face validity of content through informal feedback from community members and representatives of our project's user/stakeholder groups, along with data from expert interviews focusing on specific content areas. For face validation of the ontology – the structure of the representation itself – we are developing a structured group survey protocol designed to discover areas of consensus among experts. Together, these will provide formal indicators of the trustworthiness of the Framework as an expression of what the mathematics education community considers to be mathematics learning (Clayton, 1997; Nevo, 1985; Shavelson & Stanton, 1975).

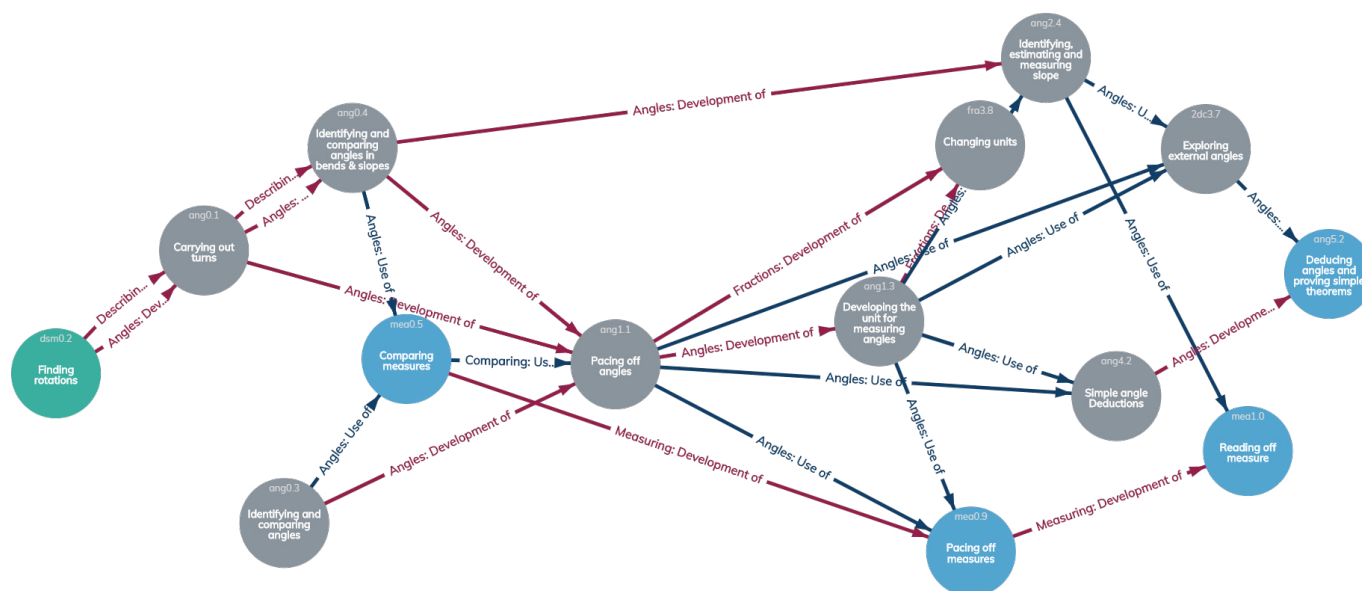
THE STRUCTURE OF THE FRAMEWORK

Our design tool brings together both knowledge of mathematics itself and a consideration of pedagogy, reflecting the multi-faceted activity that is educational design. We are aiming for a design which takes into account research and communication with teachers and designers and will make sense to them both. Other curriculum framework design projects have used similar methods but in the service of different design goals and priorities (Confrey & Maloney, 2015; Maloney & Confrey, 2013; Michener, 1978).

We treat mathematics as a connected web of ideas in which different meanings can be found at different levels of organisation. Using network graphing software, we illustrate the mathematics by a layer of different types of network nodes, and the connections between them by different types of edges. Our software allows us to build multiple layers and connections within and between these layers.

We have chosen to describe content in the Framework by student actions, adapting Malcolm Swan's framework for task design (Swan, 2014). Where there are alternative evidence-based approaches (for example with or without dynamic geometry software) we record them both. Multiple connections offer multiple pathways through the Framework.

The importance of play, early in the development of a mathematical skill or concept, allows for useful intuitions to be set up, and elementary but important properties of concepts and examples to be established (Denvir & Brown, 1986a, 1986b; Michener, 1978). As described above, we have embedded exposure to mathematical content in ways that could occur at an earlier stage than found in many curricula.



Framework features

Waypoints (Mathematical Content Layer)

The majority of our nodes are waypoints, which we define as places where learners acquire knowledge, familiarity or expertise. The specification of waypoints in our ontology is based on characterisation of learning sequences by Michener (1978) and Swan (2014, 2015). Each waypoint contains a summary of the mathematics (the “what”) and why it is included (the “why”). Waypoints at the beginning of a theme, as described above, are additionally designated *exploratory*. We recognise also that it is useful to bring different ideas together where the whole is greater than the sum of the parts – we identify these as *landmark* waypoints.

Edges

Waypoints are connected by edges. Each is labelled according to a mathematical theme, and whether the connection between the waypoints is best described as a conceptual progression, or the use of a skill or concept.

Research nodes (Research Layer)

Our design is informed by research evidence and conversations with knowledgeable others. Some decisions we make are unsubstantiated other than by the team’s own practical experience. Our decision making is transparent because we record the basis for our writing in our Research Layer which comprises our research nodes and summaries.

END NOTES

This update is written when we are just over halfway through our first period of work. By 2020, we will have put in place a complete mathematical layer to cover approximate ages 3–16, together with the relevant layers of research and glossary. We will have partially connected the Task and PD Layers so that we can share what will be possible when we have populated it all. We will also have made progress on the complicated landscape that is post-16 mathematics.

Glossary nodes (Glossary Layer)

Key mathematical terms or phrases are defined in *glossary nodes*. These allow us to access definitions while looking at the Framework content and surface the content which is linked to a particular term.

Other layers

Ultimately, we expect there to be a linked Task Layer in which the task nodes will describe either the detail of classroom activity (including formative assessment) or a summative assessment activity. We know that some of the tasks will be linked to just one waypoint while others may span two or more, including across mathematical topics. This exemplifies in a different way the interconnectedness of mathematics. The Professional Development (PD) Layer will contain PD nodes which will link to each other and to the appropriate nodes within other layers.

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