

# A heady brew: designing filtered mathematics education research documents (Espressos) for teachers

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## Introduction

### Cambridge Mathematics

Cambridge Mathematics is a collaborative organisation at the University of Cambridge, bringing together the expertise of the Faculty of Mathematics, the Faculty of Education, Cambridge Assessment, and Cambridge University Press to create and support a framework for mathematics education for all students aged 3–19. We are developing a flexible and interconnected digital framework to help reimagine mathematics education 3-19. Our design process is transparent, collaborative and research- and evidence-informed.

We think mathematics learning 3-19 can be more connected and coherent and we are providing a structure to make this happen. We are working with specialists in mathematics education all over the world and expect to have an initial version ready by 2020. The framework emphasises connections between mathematical ideas that support students in building mathematical understanding, using a structure based in the activity design work of the Shell Group at the University of Nottingham.

During our work on the Framework, immersing ourselves in the latest and most seminal mathematics education research, we wanted to create documents for teachers that would help make some of these important ideas accessible and applicable to the classroom.

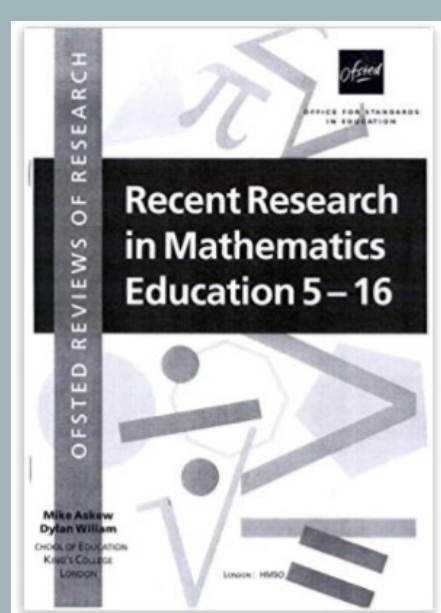
### The brief: design principles and constraints

We set up a list of design criteria (and limitations in scope) for these documents, which reflected many of the aspects of the Framework itself:

- based on as objective as possible a review of the current research literature, while acknowledging limitations of person-power, time and inevitable bias
- restricted to a two-page document
- containing an attractive and accessible well-constructed mathematical diagram
- fully referenced, with hyperlinks to original research
- carefully focused on one significant question at a time; not attempting to tackle questions too broad or nebulous
- using clear and unambiguous language; defining terms where appropriate
- using the research to clearly outline classroom implications where possible, but without oversimplifying or extrapolating beyond the scope of the research
- visually designed with a focus on aesthetics, clarity and usability for teachers and head of department/subject leaders in mathematics; requiring no object medication to be appropriate and helpful to those not necessarily previously familiar with research content and language

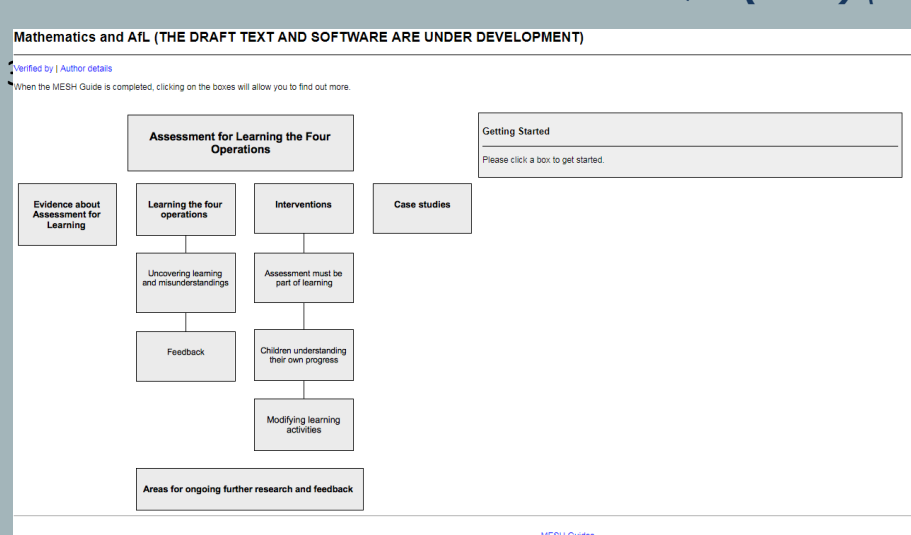
### Examples

The following are some of the documents produced by others that exemplify some (but not all) of our design principles for Espressos:

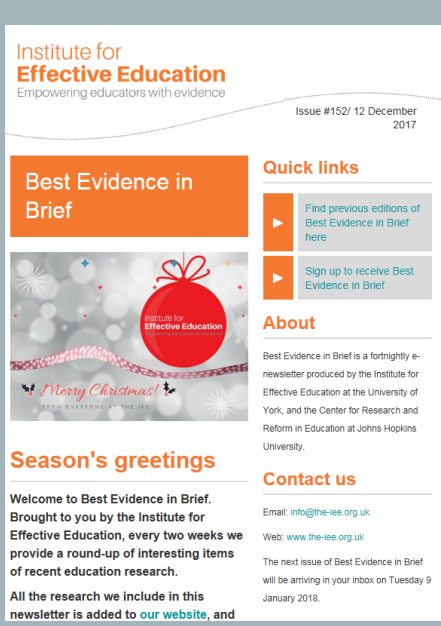


**Recent Research in Mathematics Education 5-16**  
Askew, M. & Willam, D. (1995)  
Great Britain: Office for Standards in Education

MESH guides ([www.meshguides.org](http://www.meshguides.org))  
For example, **Mathematics and AFL**  
Lee, C (2013) (draft)



**Best Evidence in Brief**  
(fortnightly from 2011)  
Newsletter  
Institute for Effective Education



See [www.cambridgemaths.org/espresso](http://www.cambridgemaths.org/espresso) for more information

**TALKING POINT:** WHAT ARE THE ISSUES IN LEARNING AND ASSESSING TIMES TABLES?

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WHICH TIMES TABLES?

PERCENTAGE ACCURACY ON TIMES TABLES TESTS KS2 PUPILS (Elementary 2015)

1. Faculty with times table tests is needed to perform higher order mathematical processes effectively. (Mason and Greville 2002) Reported systematic practice of times tables in schools and the declarative knowledge serves as a building block for procedural knowledge. This process is key to making the network of basic times tables facts for recall (Kilpatrick, Lee & Zuckerman 2002). The skills are the basis for later achievement in maths (Mang & Evans 2007; Maloney & Geary, 2005) and nearly all maths curricula in high-performing countries contain the memorisation of times tables up to 10 x 10 (Bridges & Steiner, 2005) but just learning times tables doesn't mean that a student will be good at later mathematics (Bishop & Fuchs, 2010).

**IMPLICATIONS:** Knowing times tables is important and should be taught in school at a young age to ensure fluency for later mathematics, but it is not the only or most important thing to learn for early mathematics.

2. Automatic memorisation of times tables does not ensure that a student will be good at later mathematics. (Bishop & Fuchs, 2010) However, young children with a high proficiency with working memory skills also correlate with generally high academic performance, especially in problem-solving and reasoning on paper to higher maths levels, which can have a negative impact on their achievement in maths by coping with rote learning (Bishop et al. 2013).

**IMPLICATIONS:** Maths curricula can interfere with memory, which means learning times tables may create anxiety that affects test results and undermines confidence, a barrier to working towards automaticity.

3. Studies comparing computer-based practice of times tables with pencil and paper practice (e.g. Cooley, 2001) suggest that computer-based practice is more effective, perhaps because students are more motivated.

**IMPLICATIONS:** Using ICT to learn times tables facts can be more effective and motivating than paper and pen methods.

**TALKING POINT:** WHAT IS NUMBER SENSE AND HOW DOES IT AFFECT MATHEMATICS LEARNING?

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DIFFERENT THEORIES OF HOW WE PERCEIVE NUMBER

**WHAT DO WE KNOW FROM RESEARCH THAT IS KEY TO OUR UNDERSTANDING OF NUMBER SENSE?**

Number sense is a term that is defined differently by different experts – there is particular discrepancy between the two fields of cognitive science and mathematics education (Cohen, Jordan & Fleiss, 2005). Galanter, Wang & Case (2001) suggest there is some consensus on a definition that includes fluency in estimation, ability to recognise reasonable results, flexibility when mentally computing, ability to move among representations and choose the most appropriate, (Carpenter, 2002), or Galanter, Wang & Case (2001) define it as the key to the ability of counting and quantity discrimination (language) – and conclude the link between these here. There are also the processes of the other elements. In the diagram, we use a constructivist view of number sense as the key to the ability to perceive, manipulate and understand numbers (Cohen, Jordan & Fleiss, 2005). Cohen, Jordan & Fleiss (2005) suggest that the most important aspect of number sense is the ability to understand the relationship between the whole and its parts, and to understand the relationship between the whole and its parts. The building block approach to number sense is the most important aspect of number sense. The building block approach to number sense is the most important aspect of number sense. The building block approach to number sense is the most important aspect of number sense.

**IMPLICATIONS:** Number sense is not a term that all researchers agree to use in the same way, but it includes both counting and comparing skills and the basic ability to compare and represent numbers. It can be improved through teaching.

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**TALKING POINT:** A TRANSITION OR A COLLABORATION?

**A 'Transition' orientation**

A 'Collaborative' orientation

1. There are a number of models that have been applied to teaching mathematics that broadly fit with a 'traditional or progressive' framework. The key element that defines such models appears to be the 'teacher centredness' of the pedagogical approach. These include transmission or constructivist (Lowe, 2000) traditional or progressive (Lowe, 2000) models where the teacher is the primary source of knowledge, and the student is the recipient. Another description is 'teacher centred' or 'teacher centred' (Lowe, 2000). These models tend to present the two descriptions on opposite ends of a continuum (Lowe, 2000). These have been used to describe teacher beliefs in studies other than classroom practice (Swain and Swain, 2010), which brings the apparent problem of a distinction between espoused theory and theory in action (Parsons et al. 2014).

**IMPLICATIONS:** Calling a model either 'traditional' or 'progressive' and using terms may oversimplify a complex issue. They may also be misleading in terms of describing and describing practice.

2. The terms 'traditional' and 'progressive' and their related concepts can be applied to build various teacher/school/country/curriculum/assessment/teaching practice, or assessment. Most research currently focuses on teacher beliefs and attitudes. There is evidence to provide a separate perspective on teacher beliefs and practice using self-reported teacher characteristics into three broad categories – corresponding to 'traditional', 'progressive' and a third position on the spectrum using a model of traditional and progressive teaching (Parsons et al. 2014).

**IMPLICATIONS:** Considering statements about classroom practice may be less helpful to teachers to build their own beliefs and attitudes. Considering statements about a spectrum between 'teacher-centred' and 'student-centred' practice may help to inform and improve teaching.

**TALKING POINT:** HOW CAN MATHEMATICS TEACHING BE MEASURED?

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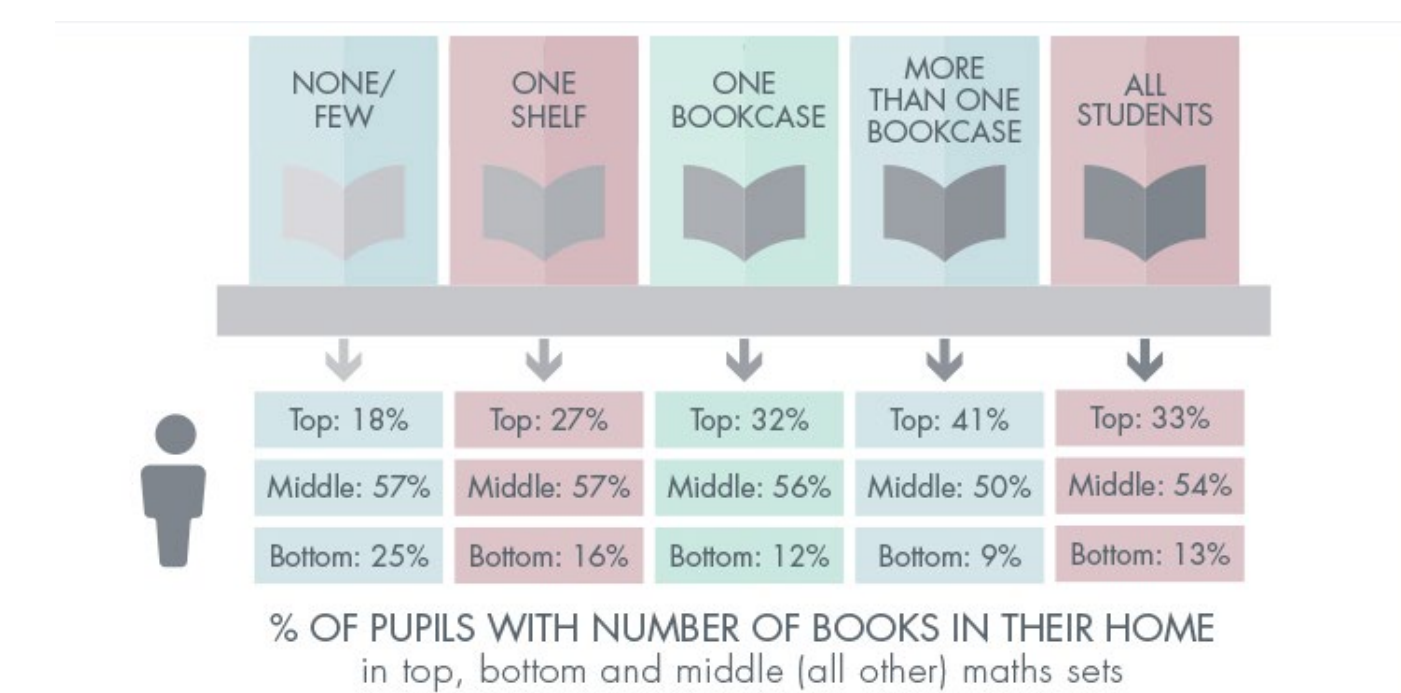
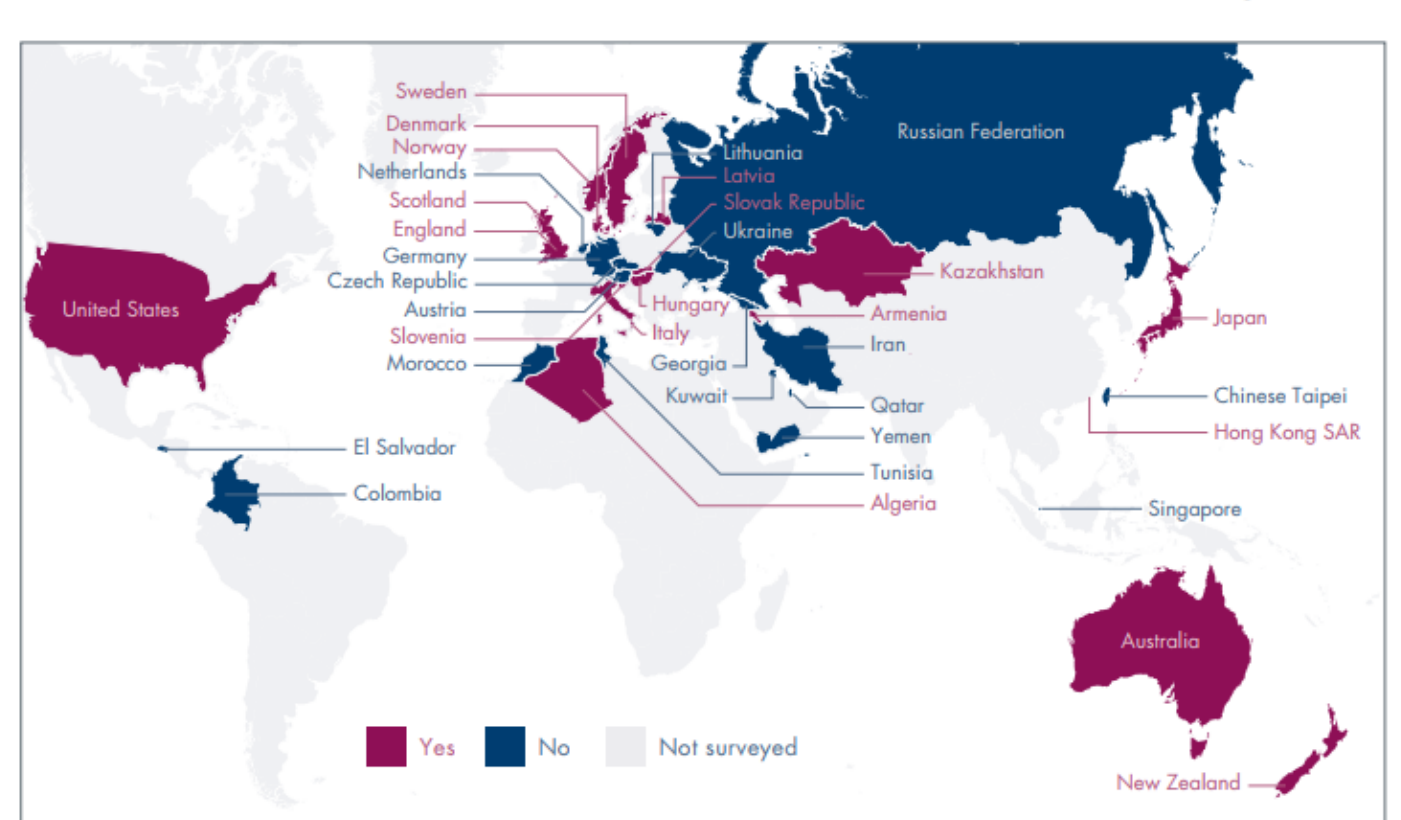
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1. There are many ways to measure mathematics teaching, including lesson observations (both with and without an instrument), student attainment (including self-reported), teacher self-reports, and other measures of good mathematics teaching (Lowe, 2000). The most common and stable measure of good mathematics teaching is student attainment. The best available evidence is provided by looking at lesson plans, student work, marking, and self-reports by teachers. Some studies show that self-reports by teachers are the most accurate measure, whereas others show that self-reports by students are the most accurate measure. It is difficult to change the effect of school, teacher and area group. It is difficult to change the effect of school, teacher and area group. It is difficult to change the effect of school, teacher and area group.

**IMPLICATIONS:** Good maths teaching makes a difference but it is hard to measure. Lesson analysis and self-reports by teachers are not effective measures of mathematics teaching. The most stable measure of effective maths teaching is a combination of observations, attainment and student self-reports.

Do countries mention calculators in their national curriculum at age 10?



## Design processes

### Collaborative effort

Cambridge Mathematics Espressos are written and edited on an approximately monthly schedule with all members of the Cambridge Mathematics team contributing to the final edited version.

- Processes include:
- Finding a topic area that we have been researching as part of the framework development that might be of interest and use to teachers and agreeing a focused research question/talking point
  - Finding suitable data concepts for the infographic (related to question) and send to the designer
  - Reading around the subject – keyword search, discussion with team and known research experts around key papers
  - Finding two or three interesting short quotes related to the research question
  - Writing first draft of numbered elements along with implications with footnotes referring to papers
  - Sending draft to colleagues to read and feedback; discuss content and structure in dedicated meeting
  - Sending versions of infographic to colleagues and discuss which is best/any amendments
  - Producing a second draft of full text with references and proof-reading
  - Sending text to designer to produce final version and making small corrections and amendments where necessary to fit design
  - Adding hyperlinks to designed version and checking
  - Uploading to Cambridge Maths site and promoting on social media and through other channels (eg newsletter, guest blogs, articles)

### User feedback

Presenting at mathematics education and teacher conferences on this topic has led to several [pieces of important user-generated feedback, including:

- Moving the 'In summary' box to the beginning of the document
- Changing the citation style from bracketed references to endnotes
- Ensuring as many original research documents as possible are not inaccessible to teachers without institutional access
- Ensuring each Espresso has a comments section on our website where users can ask for further information or make a query

We have also discussed these documents informally with a number of researchers whose work is included therein, and received some useful and positive feedback.

### Questions and next steps

Feedback from organisations such as the National Centre for Excellence in the Teaching of Mathematics and the Mathematical Association and Twitter engagement suggest the Espressos are being well received and that teachers are using them for CPD in mathematics.

- Next steps include:**
- a systemic review of downloads/read of Espressos and analysis of the data
  - a mechanism for more detailed feedback after users have opened the document (questionnaire)

- We would like to consider the questions:**
- Who is reading Espressos?
  - How are they using them?
  - How could we further ensure they are seen by mathematics teachers?
  - How can we ensure the research questions chosen are useful to the community of users?

University researchers are beginning to show an interest in collaboration with Cambridge Mathematics on these documents and work has begun on these special edition 'Espresso Doppios'.



### References

- Centre for Research in Mathematics Education, 2015. *Framework for Designing Classroom Activities*. Shell Centre, University of Nottingham
- Best Evidence in Brief: fortnightly newsletter from the Institute for Effective Education (<http://www.beib.org.uk/>, accessed 10/12/17)
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- Askew, M. & Willam, D. (1995) *Recent Research in Mathematics Education 5-16*. Great Britain: Office for Standards in Education