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# TALKING POINT:

## HOW CAN ALGEBRAIC THINKING BE EFFECTIVELY INTRODUCED?

## IN SUMMARY

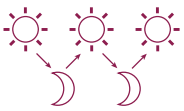
- Algebra is difficult to define and different definitions will determine how and when algebraic thinking is introduced to students
- Generational, transformational and meta-level activities are possible categories of algebraic thinking
- It may be better to teach algebraic thinking in a less discrete manner than some curricula suggest
- Introducing algebraic thinking early in the mathematics curriculum can be effective, as young students may be more successful at thinking algebraically than previously thought
- Introducing algebraic thinking can be done through the general to the particular or vice versa
- Approaching algebraic thinking through developing students' notion of equivalence is effective
- Using technology can enhance algebraic thinking by providing more contexts and representations

Algebraic pattern structures in early children's literature (adapted from Elliott et al, 2005)

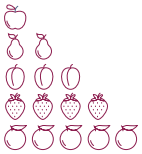
TVHC*

| | | |
|---|---|---|
| Natural numbers | Mon | 1 |
| | Tue | 2 |
| | Wed | 3 |
| | Thu | 4 |
| | Fri | 5 |
| Pattern breaking | Sat | 10 |
| | Sun | 1 |

Cyclic



Triangular numbers



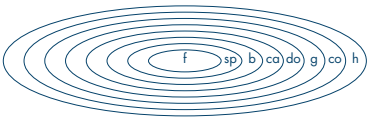
TWAOLWSAF**

| | |
|---|---|
| Natural numbers | Ol-f |
| | Ol-f-sp |
| | Ol-f-sp-b |
| | Ol-f-sp-b-ca |
| | Ol-f-sp-b-ca-do |
| | Ol-f-sp-b-ca-do-g |
| Triangular numbers | Ol-f-sp-b-ca-do-g-co |
| | Ol-f-sp-b-ca-do-g-co-h |
| | Ol-f-sp-b-ca-do-g-co-h- |

Subsets

Ordering

f<sp<b<ca<do<g<co<h




*The Very Hungry Caterpillar by Eric Carle **There Was An Old Lady Who Swallowed A Fly by Simms Taback

1

Algebra is difficult to define and how early it should be introduced depends on which definition of concepts you focus on – it may be better to ask what 'ideas, logic, techniques, and habits of mind' algebra encompasses, and then consider which of those can be learned early without losing 'intellectual integrity' and maintaining a 'coherent' whole'[1]. Sutherland (1997) defines algebra as being comprised of three main areas: '*generational activities* – discovering algebraic expressions and equations; *transformational rule-based activities* – manipulating and simplifying algebraic expressions, solving equations, equivalence and *global, meta-level activities* – ideas of proof, mathematical structure and problem solving'[2]. In most school curricula, algebra is a separate 'strand'; however in some more recent research-based approaches, such as RME, in both primary and secondary school there are no separate courses in algebra[3].

**IMPLICATIONS:** Algebra is difficult to define, even among mathematics education researchers

Defining what one means by 'algebra' will affect how and when algebraic thinking is introduced to students

One definition of algebraic activity describes generational, transformational and meta-level categories

Algebra is often taught separately from other areas of mathematics in current curricula but some research suggests integration is effective

'The adolescent who has mastered algebraic concepts has gained a vantage point from which he sees concepts of arithmetic in a broader perspective.'

**Vygotsky**

'In the 1990s, enrolment in algebra was characterized as "The New Civil Right" by Robert Moses. The Clinton administration declared completion of an algebra course by the end of eighth grade a national goal.'

**Tom Loveless, Brown Center Chalkboard**

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2

It has been a common assumption that arithmetic should be taught before algebra, but some research shows that 'even children in primary school exhibit algebraic thoughts…and can be successful in solving algebraic problems…introducing tasks that require algebraic thinking early may overcome problems[4]. Recent research suggests an early start in algebra is 'not only possible but necessary, and that younger children can do much more in this area than was previously thought,'[10] including using formal language[8]. Introducing algebra 'late, abruptly, and in relative isolation' is regarded as ineffective[10].

**IMPLICATIONS:** Introducing algebraic thinking early in the mathematics curriculum can be effective
Young students may be more successful at thinking algebraically than previously thought

3

There are, broadly speaking, two ways to facilitate algebraic thinking: from the general to the specific (eg Davydov, 1975[5]) and from the specific to the general – this last approach is often built into contemporary school curricula, where 'children are expected to make their own generalisations based on seeing multiple examples of number problems'[6] which involves some 'considerable adjustment' to thinking[7]. A suggested way to effectively navigate this approach might be to introduce primary-age pupils to the idea of variable and quasi-variable by considering invariance in number sentences[10]. By contrast, theories that suggest early introduction of algebraic thinking through the general are often focused on early conceptions of *comparison* with young children, leading to ideas such as the commutative property – all without any need to 'measure' or define the specific quantities involved[8]. The Gattegno-Cuisenaire method demonstrates that very young children can 'master algebraic expressions' by using coloured rods to reason with equivalence and proportion, where 'number names for the rods are not introduced until their relationship to one another has been fully explored symbolically'[9].Another way to describe these two approaches might be 'thinking with the unknowns' vs 'thinking with the knowns and working from these to the unknowns'[3].

**IMPLICATIONS:** Competing theories suggest the introduction of algebraic thinking can be done through the general to the particular or vice versa
One suggested approach, beginning with the general, is through abstraction to quantity and then finally number, perhaps using Cuisenaire rods
If starting with the particular, research often suggests examining simple arithmetic properties and drawing out the concept of (quasi-) variables first

4

More recent research suggests there should be more emphasis on the 'emergence and development of students' *notions* of equivalence' rather than just teaching students how to maintain equivalence, as well as a move away from just performing transformational algebraic activity towards considering its 'structural underpinnings'[10].
A fostering of the development of 'algebraic thinking' – defined here to include deliberate generalisation and reasoning with those generalisations – is suggested as opposed to 'the teaching and learning of specific bits of algebra content'[11]. Using technology to teach algebraic thinking allows students to work with algebra in a variety of rich and meaningful contexts as well as facilitating multi-representational activities[10].

**IMPLICATIONS:** Approaching algebraic thinking through developing students' notions of equivalence is effective
A focus on a connected development of algebraic thinking rather than just isolated algebra content is suggested
Technology can increase access to rich contexts for algebraic thinking as well as a variety of representations for students

Lucy Rycroft-Smith, 2018

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