

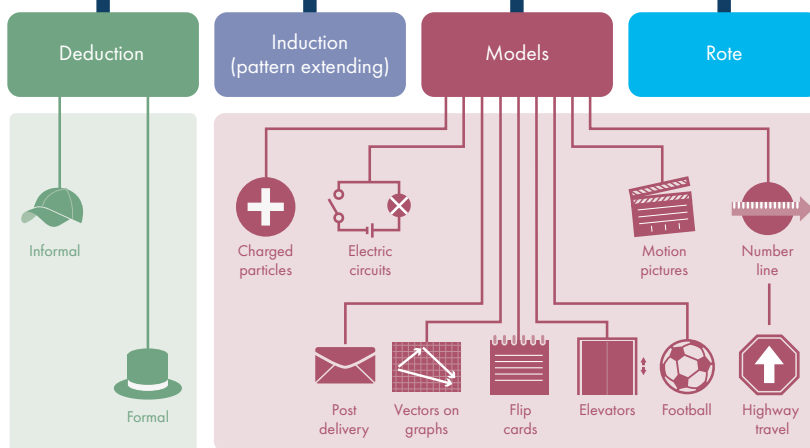
TALKING POINT:

WHAT DOES RESEARCH SUGGEST ABOUT EFFECTIVE WAYS TO INTRODUCE NEGATIVE NUMBERS?

IN SUMMARY

- The idea of negative numbers may feel odd and counterintuitive to students
- Students need to have a perception of number as both counting and measurement, as preparation for learning about negative numbers
- An early focus on subtraction and multiple concepts of zero is important
- Acknowledging negative numbers when teaching subtraction may be effective, but waiting until students have a sense of the structure of number systems to explore them more fully is suggested
- Students may be confused between negative numbers, decimals and fractions
- Teachers should carefully consider their language around negative numbers, especially the use of the words 'minus' and 'negative'
- There are many useful models and metaphors for teaching negative numbers, but no one is totally representative on its own and all have limitations
- Students should understand the minus sign in at least three senses: unary, binary and symmetric

Strategies for teaching multiplication of negative numbers



Adapted from Peterson, *The Arithmetic Teacher*, copyright 1972, by the National Council of Teachers of Mathematics (all rights reserved) and Arcavi & Bruckheimer (1981)

1 The inclusion of negative numbers in school curricula is 'surprisingly modern'¹ and even serious mathematicians, when first faced with them as a concept, described them as 'absurd' and 'nonsensical'². Dealing with negative numbers is particularly difficult² as they present significant conceptual contradictions for pupils, who may have to reconceptualise their understanding of number to accommodate them.³ Working effectively with them means students must understand number as both counting and measurement.⁴ They are also most students' first encounter with fictive numbers (numbers that can't be modelled with physical objects)⁵. There is a relatively sparse amount of research on learning integers (positive and negative whole numbers)³, given how central it is to mathematics curricula.

IMPLICATIONS: Negative numbers represent a significant conceptual shift for students

The idea of negative numbers is a relatively recent one and may feel counterintuitive to students, as they are fictive numbers

Students need to understand number as both counting and measurement in order to incorporate negative numbers into their understanding

'No good model has yet been found which embodies, in an obvious way, all the properties of integers'

T. Leddy, 1977

'What then when I realized that no one could explain to me how it is that a minus times a minus equals a plus?'

Stendhal, 1890

2

Even young children (7-8 year-olds) are able to solve simple contextual tasks involving negative numbers in words, but not using formal notation⁴. However, researchers do not currently agree on the question of when and how to introduce negative numbers; some suggest an understanding of mathematical structure should be in place first⁶, whereas others point to a need for acknowledging or exploring negative numbers as part of early ideas such as subtraction, as a way of dealing with possible misconceptions rather than perpetuating them⁷. An early focus on number sense, especially subtraction and multiple concepts of zero, is suggested as groundwork for learning about negative numbers⁸.

IMPLICATIONS: Delaying the introduction of negative numbers may contribute towards unhelpful misconceptions later on, but an awareness of mathematical structure is an important pre-requisite

Spending significant time on subtraction and the multiple concepts of zero in early primary education is advised

3

Students have to contend with three particular issues when learning about negative numbers: understanding the meaning of a numerical system, with both direction and magnitude incorporated into the concept of 'number'; the meaning of arithmetical operations with negatives; and the meaning of the minus sign itself². Confusion between decimals, fractions and negatives is common. The language associated with negative numbers – for example 'minus 5' versus 'negative 5' can be difficult for students to navigate¹⁰.

IMPLICATIONS: Without a clear concept of number as a system of both direction and magnitude, students may struggle to deal with negative numbers

Students may be confused between negative numbers, decimals and fractions when first meeting them

A careful consideration of language (eg 'minus'/'negative') around negative numbers is recommended

4

There are multiple models for helping to introduce and teach negative numbers (see infographic for some related to multiplication of negative numbers), but no single one is wholly metaphorically representative¹ and many studies have tried, without much success, to determine which models might be most useful to students⁵. There is a tension between justifying and explaining negative numbers by internal consistency (symbolic ideas) and justifying them by realism (concrete ideas)³, but it is important to use real-world examples where possible². For a full understanding of negative numbers, students must be able to interpret the minus sign in at least three ways: unary (knowing the sign makes the number after it negative), binary (the action of taking away) and symmetric (taking the opposite of a number)⁵. Humans seem to process negative and positive numbers differently; there appears to be a negative spatial number line model present in the minds of most people¹¹, which suggests use of number line models in the classroom are effective. It is important for teachers to understand limitations and conditions of use for models/metaphors, as they may help initially but extending them or using them inconsistently may also cause confusion⁸.

IMPLICATIONS: There are many useful models and metaphors for teaching negative numbers, but no one is representative on its own and all have limitations

Teachers may use logical deduction or an appeal to examples (or both) to explain negative numbers, but there is a tension between them

Students should understand the minus sign in at least three senses: unary, binary and symmetric

Isolated working memory training may not lead to improvements in mathematics attainment

Lucy Rycroft-Smith & Tabitha Gould, 2018

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