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TALKING POINT:

WHAT DOES RESEARCH SUGGEST IS IMPORTANT FOR STUDENTS WHEN DEVELOPING CONCEPTS OF THE MEAN?

IN SUMMARY

- Allowing students to develop early, informal experiences of the mean may help them develop better conceptual understanding and reasoning skills rather than learning the procedure by rote first
- The mean is complex and should not be oversimplified or taught formally too early; it may be less intuitive to students than the median or mode
- It is important that the mean be considered alongside other measures like shape and spread and placed within the bigger picture of a statistical toolkit rather than overemphasised
- There are multiple models to aid in conceptual understanding of the mean such as a balance point, fair share and ratio model, which students should have the opportunity to consider; it may be useful to start with the fair share model as the simplest
- Zero values and the context of the data are important to consider alongside calculation of the mean
- The mean is an example of an abstract mathematical construction and time is required for students to explore it deeply
- Students should have the opportunity to discover that the mean is useful in comparing groups of unequal size when appropriate but that it is not always a typical value

AN INTEGRATED CONCEPTION OF THE MEAN

The mean

- The mean is located between the extreme values
- The sum of the deviations from the mean is zero
- The mean is influenced by all values in the data set
- The mean does not necessarily equal one of the values that were summed
- The mean can be a fraction that has no counterpart in physical reality
- When calculating the mean, a value of zero, if it appears, must be taken into account
- The average value is representative of the values that were averaged

Adapted from Strauss & Bichler (1988)

1

The arithmetic mean is complex. It may be less intuitive to students than the median or mode[1] and students are not often offered good opportunities to construct informal intuitive understanding before more formal learning happens[2]. Mathematics curricula frequently represent averages, particularly the mean, only as a single summary of groups of values or simple locations in distributions[1] instead of a thoughtful combination of models such as balance points, fair shares, and as a ratio of per-one member of the group. Students benefit from experiences of the mean which are more fluid, exploratory and less theory-laden[3,5]. First encountering the mean as an algorithm only, or learning it by rote, may cause a short-circuit in reasoning for students[2,4].

IMPLICATIONS: The mean is complex and may be less intuitive to students than the median or mode; it may be oversimplified in resources as a single summary or location

Allowing students to experience fluid and exploratory ideas about the mean may help them develop better conceptual understanding and reasoning skills

2

Research into the teaching of the mean has been focused on student understanding of measures of centre without also considering variability[5]. Because the mean is just one option of many for exploring data[6], it is important to consider general features like shape and spread alongside it, and look at distributions in numerous ways[1]. Like almost all statistical measures, the mean captures group properties and stabilises as more data is collected and it is important to consider the mean as part of the overall picture. Statisticians see 'group features like median and mean as indicators of stable properties of a variable system... that only become evident in the aggregate'[1]. The distribution should be considered alongside the mean to explore whether the mean is typical or appropriate (for example, when the data are not symmetrical).

IMPLICATIONS: It is important for students to consider the mean alongside other measures like shape and spread

Placing the mean within a bigger picture of statistical uncertainty allows students to consider it as part of a toolkit rather than overemphasising it

It is important to challenge ideas of the mean as always typical and to ensure students think about features of the distribution such as symmetry when considering appropriate measures

3

There are several models to help students understand the mean: it can be modelled as a 'mathematical point of balance' – i.e. an indicator of centre, for summarising, comparing and describing data sets. The difficult idea that locating the mean is a process of balancing deviations of data away from it, especially when the data are not symmetrical, is important for successfully understanding the structure of the mean[10]. Balance is also used in the sense of 'compensation' to indicate taking a sample and then 'scaling up'[7]. Another way to model the mean is as a 'fair share', which may be more effective for younger learners[2]. A different but useful model for the mean is the ratio perspective: thinking about the mean as the reverse of 'multiplying up' contributing units to make an accumulative total. This allows students to associate the mean also with dividing, thinking about a ratio representing per-one contribution to the total[12]. An important part of this idea is taking into account zero values, if they occur, which can be one of the hardest aspects of the mean to understand (see infographic)[8], and also considering and identifying outliers[9], which means an understanding of context is important[10].

IMPLICATIONS: Presenting students with multiple models of the mean – such as a balance point, fair share and ratio model – allows them to develop their conceptual understanding and appreciate its complexity

Younger students may find the fair share model useful to consider to begin with

Students should have opportunities to consider the importance of zero values and the meaning of the context in which they are calculating the mean

4

Although the calculation required to perform the mean can be relatively straightforward, it is a mathematical construction that expresses a relationship and so requires a level of abstracted thinking to understand; students of age 10–14 often have a difficult time assimilating all its properties[10]. Conceptions of the mean should also be multiplicative, which allows the comparison of groups that might not be the same size when thinking about an attribute that is sensitive to group size[11,12]. Concepts such as a weighted mean can be particularly complex for students to understand[10].

IMPLICATIONS: Recognising the usefulness of the mean in comparing groups of unequal size allows students to consider its affordances and also limitations

The mean is likely to be one of students' first encounters with a mathematical construction to express a relationship therefore they need time to explore it thoroughly

'Although most people know the procedure for finding a mean set of values, the mathematical relationship itself remains opaque'

Mokros & Russell, 1995

'The usefulness [of] the mean is often as a base for further calculation – estimated... cost or effect of a change, for example. If we wanted to calculate how much it would cost to give all employees a 10% hourly pay increase, then this could be calculated from mean earnings (multiplied back up by the number of employees)'

House of Commons Research Briefing, 2007

Lucy Rycroft-Smith & Darren Macey, 2019

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