

TALKING POINT:

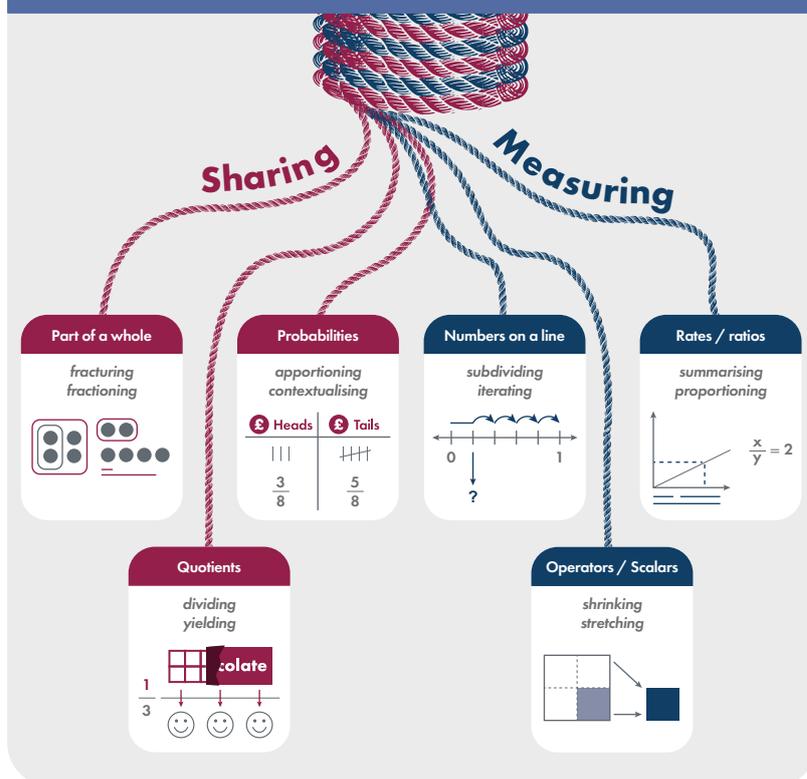
WHAT DOES RESEARCH SUGGEST ABOUT EFFECTIVE WAYS TO INTRODUCE FRACTIONS?

IN SUMMARY

- Seeing fractions as a set of interrelated concepts encompassing both dividing and measuring may help students to understand them
- Considering fractions as abstract and composite ideas can help extend students' understanding from natural numbers to real numbers
- Early understanding of fractions is supported by work in equipartitioning (fair sharing) which forms part of the idea of division and may help support counting development
- It is important to focus on one half as a key mathematical idea and to recognise that sharing in even quantities is usually found to be easier; circle models are particularly difficult
- The number line is a key representation for developing understanding of fractions, but multiple representations are important
- Exploring ideas of unitising (grouping) and norming (naming the reference unit) is an important part of understanding fractions
- Students should have opportunities to see fractions as relative amounts and as abstract ideas manifesting in different but equivalent ways

FRACTIONS

A set of intertwined constructs



1

Fractions are complex and composed of multiple meanings¹ and not understanding them could be one of the most serious obstacles to children's mathematical advancement². 'Early fraction ability emerges only slightly later than ability to deal with whole numbers, but both are present at some level by age four.'¹ Working with fractions entails an important shift in thinking as part of the process of extending ideas of natural numbers to real numbers³. Students need to be able to make a transition from just thinking about specific instances of fractions (with links to strong visual remembered images like shaded-in semicircles⁴) to 'knowing' fractions in a more holistic and abstract sense⁵. Students benefit from meeting fractions in the contexts of, and linking between, dividing and measuring as they do not always transfer understanding from one situation to the other⁶.

IMPLICATIONS: If students see fractions as a set of interrelated concepts, with multiple uses and models, they may be able to manage their complexity more successfully

Fractions are a key concept in extending children's understanding of number from natural numbers to real numbers

Just as with natural numbers, a conception of fractions as existing as specific instances (with visual representations) should build into an abstract, composite and general notion

2

The first steps in understanding fractions come from equipartitioning (fair sharing) and comparing and discriminating which of two quantities is larger. Significant numbers of preschool children can successfully divide quantities into equal sub-units⁷; through this equipartitioning (partitive) division cognitively precedes multiplication⁷. Teachers can do sharing activities with children even before they have become secure at counting, because children can use dealing or mental recording strategies very early. One half is a crucial mathematical concept; 'how to stimulate and deepen the child's conception of one half is at the core of effective mathematics education'³. When dividing into equal pieces, students consistently accomplish 2- then 4- and commonly 8-splits before others; 3-splits are difficult, especially on circular shapes⁷. The use of circular models can be a problem when sharing because they are difficult to cut and to interpret.

IMPLICATIONS: Early understanding of fractions is supported by work in equipartitioning (fair sharing) which forms part of the idea of division

Children can use dealing or other strategies to share, considering what 'fair' means, very early on – and this may help support counting development

It is important to focus on one half as a key mathematical idea and to recognise that sharing in even quantities is usually found to be easier than in odd ones

Children may find use of circular shapes to model sharing difficult; multiple representations may support more flexible thinking

3

Students who have an understanding of fractions as shares often do not have an understanding of them as magnitudes (numbers with a particular size)⁶. Developing students' conceptions of one half (and from there, other fractions) from a qualitative idea (estimating) to a quantitative one (counting) is an important goal³. The number line is a key model for representing fractions because it suggests both subdivision and iteration and because it may help to highlight symbolic aspects, as it requires at least two reference points to become numerically meaningful⁸. Using a number line can also support development of the subconstruct of fraction as an operator, in which, for example, $\frac{3}{4}$ is interpreted either as $3 \times [\frac{1}{4} \text{ of a unit}]$ or $\frac{1}{4} \times [3 \text{ units}]$ ⁹.

IMPLICATIONS: The number line is a key model for developing an understanding of fractions

Measuring is an important model to help refine understanding of fractions from estimating quantity to counting

It is important to use a combination of representations to help combine ideas of fractions as shares and as magnitudes

4

Using fractions is strongly related to unitising: 'How many or how much is meant by $\frac{1}{2}$ is ambiguous unless we know (among other things) to which whole it represents. Norming and unitizing are crucial to fraction concepts.'¹⁰ Fractions are different from natural numbers in that they are relative rather than a fixed amount – the same fraction might refer to different quantities and different fractions may be equivalent¹¹. Exploring these ideas is the beginning of a sound understanding of proportional reasoning¹².

IMPLICATIONS: As part of learning about fractions, students should explore ideas of unitising (grouping) and norming (naming the reference unit)

Having opportunities to explore fractions as relative amounts and as manifesting in different but equivalent ways – in other words proportional reasoning – may help to provoke deeper understanding and lay groundwork for later mathematics

'The complicated semantics of fractions is, in part, a consequence of the composite nature of fractions. How is the meaning of 2 combined with the meaning of 3 to generate a meaning for $\frac{2}{3}$?'

Ball, 1993

'In the Early Years... learners' school experience is limited to pizzas, sticky paper and chocolate! Do the children have experience of objects, shapes and amounts in equal measure and do they have experience of the whole being something other than 'one'?'

NRICH, 2013

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