WHAT DOES RESEARCH SUGGEST ABOUT THE DEVELOPMENT OF PROPORTIONAL REASONING IN MATHEMATICS LEARNING?

TALKING POINT:

• Proportional reasoning involves an understanding of proportionality – change and invariance in relationships – and should be integrated and connected across mathematical strands

• Allowing students a wide range of proportional reasoning experiences over a number of years, beginning when they are young alongside development of their rational number sense, is recommended

• Proportional reasoning is more than just finding missing values; it is a lens for problem-solving that lays important foundations for algebraic thinking

• Premature memorisation of rules is likely to inhibit development of proportional reasoning

• Students should have opportunities to sketch, describe and represent proportion problems and relationships between quantities in informal, invented ways before moving towards symbols and algebra

• Students should be encouraged to explore different areas of mathematics (including slope, scale, probabilities, vectors and rates) through a lens of proportional reasoning, finding the connections and similarities

IN SUMMARY

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3. Proportional reasoning is more than just finding missing values; it is a lens for problem-solving that lays important foundations for algebraic thinking

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5. Students should have opportunities to sketch, describe and represent proportion problems and relationships between quantities in informal, invented ways before moving towards symbols and algebra

6. Students should be encouraged to explore different areas of mathematics (including slope, scale, probabilities, vectors and rates) through a lens of proportional reasoning, finding the connections and similarities

IMPLICATIONS: Proportional reasoning is key to mathematics learning, involving an understanding of change and invariance

Proportional reasoning should be integrated and connected across mathematical strands such as number, geometry, probability and statistics

Students should have a wide range of proportional reasoning experiences over a number of years in order to develop proficiency in its use
Proportional reasoning develops alongside rational number sense and is rooted in early (pre-school) experiences such as comparing, measuring, building up, sharing, growing and shrinking. It also relies on an understanding of dividing (splitting into equal parts) and unitising (measuring several equal parts as one unit). Exploring problems through a lens of proportional reasoning allows pupils to form a framework for them to think about and justify relationships, which may help prevent numerical computations “materializing from nowhere.” Careful choice of values used in problems may help provoke a need for multiplicative reasoning. Teachers have a tendency to over-interpret students’ responses and judge that they have understood proportionality on the basis of a correct answer. Premature use of “shortcut” rules may form a barrier to the development of proportional reasoning as students may apply them without thinking — because teaching of proportional reasoning is often limited to memorising procedures for solving missing value problems.

**IMPLICATIONS:** Young children can begin to explore early ideas of proportional reasoning alongside the development of rational number sense, building on dividing and unitising.

Successful proportional reasoning is not necessarily demonstrated by a single correct answer. Memorisation of rules may encourage students to apply rules without thinking and therefore inhibits development of proportional reasoning.

**IMPLICATIONS:** Proportional reasoning is a key part of the basis for algebraic thinking and the two should be explicitly linked for students.

When considering proportional problems, students should have opportunities to explore and represent the problem situation and the quantities involved with manipulatives and diagrams before trying to model the situation using symbols or formal methods. Routinely asking students to identify the quantities that are constant, invariant, or that covary, and describing how the relationships between quantities change using descriptive language, as well as encouraging them to model relationships mentally and draw diagrams, is important. Allowing students to see, in a sustained way, proportional relationships across varied contexts such as slope, scale, probabilities, vectors and rates can be particularly effective.

**IMPLICATIONS:** Students should have opportunities to represent proportion problems and quantities in informal, invented ways before moving towards symbols and algebra.

Giving students frequent opportunities to describe and sketch quantities that change together in different ways is an important feature of proportional reasoning.

Students should be encouraged to explore different areas of mathematics (including slope, scale, probabilities, vectors and rates) through a lens of proportional reasoning, seeing the connections and similarities.