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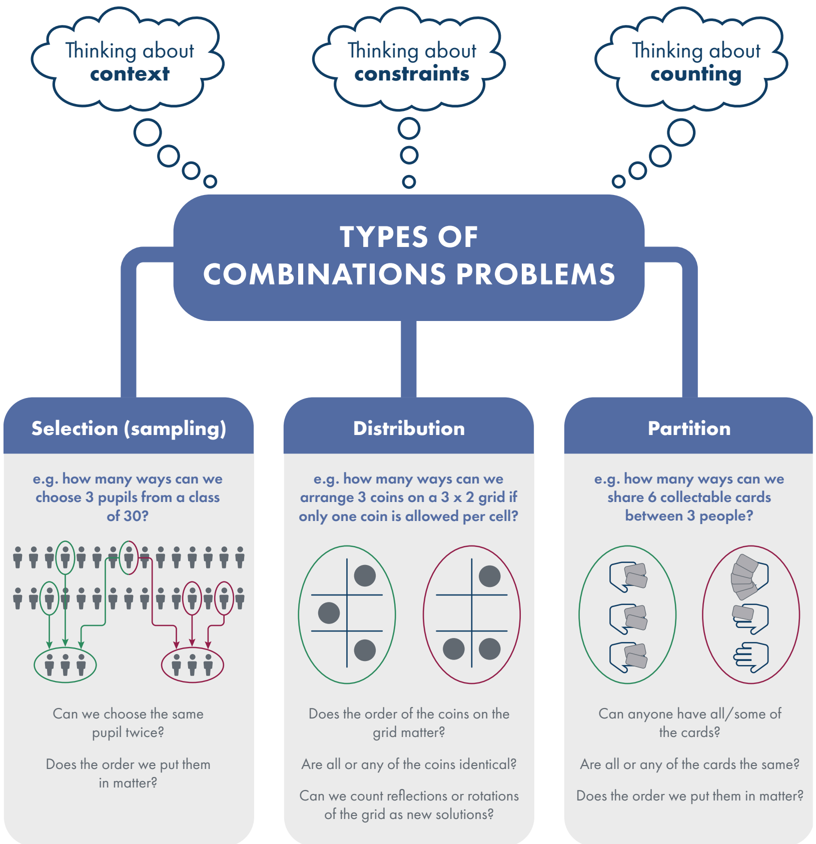
TALKING POINT:

WHAT DOES RESEARCH SUGGEST AROUND THE INTRODUCTION OF COMBINATORICS (OFTEN CALLED COMBINATIONS)?

IN SUMMARY

- Early combinatorics, often simply called *combinations*, is an accessible mathematical topic even at the primary level, because it is based on simple but important ideas of counting, especially the counting unit
- Using the *fundamental counting principle* and asking important questions about what to count and how to represent the count is important in early development of combinations
- Combinations-type problems can often be rooted in meaningful and real-life contexts and this, supported by the use of manipulatives and representations such as lists, diagrams and tables, can allow students to solve them more effectively
- Combinations problems support development of and connections between varied mathematical concepts
- Solving combinations problems particularly supports the development of logic and early proving
- Students should be encouraged to work towards using pattern and structure when solving combinations problems, eventually being able to work systematically
- Combinations problems may allow students multiple opportunities to begin to conjecture and to generalise, supporting later work on relationships and functions



1

Combinatorics (which, at this level, is often called *combinations*) involves choosing, arranging and counting objects in sets and has been described as one of the most interesting, versatile and accessible mathematical topics, particularly well-suited to problem solving, although rarely recognised as such.[1] Combinations problems can be suitable for all ages, including very young children (for instance, "In how many ways can three toy bears sit on a sofa?")[2] as they prompt students to consider fundamental enumeration and computation questions such as: what to count; how to make sure things are counted exactly once; and how to order things in meaningful ways so as to count them.[3] Asking questions such as "Are all the items identical?", "Can I repeat an item?" and "Does order matter?" – along with a careful consideration of context – allows students to define exactly what to count in the problem, which is necessary to solve it.[4] Early combinations work is based on an understanding of the *fundamental counting principle*[5] (also called finding the Cartesian product [6]): if there are *n* ways of doing one thing and *m* of another, there are *nm* ways of doing both.

IMPLICATIONS: Early combinatorics, often simply called combinations, is a versatile and accessible mathematical topic which can be taught across age ranges, even to very young students

Combinations problems require resolving important questions such as what to count and how to count them exactly

The *fundamental counting principle* is an important foundation for developing ideas about combinations

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Combinations problems are an important part of the experience of primary-age pupils (and beyond) because: they allow for solving a variety of real world problems; manipulatives and other representational tools can be used to solve them;[5] they are often low-threshold, high-ceiling; they promote mathematical thinking skills; they can help the development of many concepts, such as equivalence, order relations, samples, and functions; and they structurally connect ideas across many mathematical topics.[7] Researchers have defined three different types of combinatorics problem models: *selection*, *distribution* and *partition* (see infographic) which relate to early concepts of sampling, sharing and ordering;[7] combinatorics provides a rich and varied source of contextual problems for students of all ages to work on that connects many important mathematical ideas. A good understanding of combinations is essential for understanding probability,[7] and is also fundamental to early development of logical thinking[8] and ideas used in proving, such as making conjectures, generalisation and systematic thinking.[7]

IMPLICATIONS: Combinations problems provide meaningful contexts, are accessible, and allow for the use of manipulatives to solve them

Combinations problems support development of many varied mathematical concepts such as probability, samples, and functions, and making connections between them

Solving combinations problems supports the development of logic and early proving

3

Young children use manipulatives and a variety of pictographic, iconic, and symbolic representations in drawings, lists, tables and diagrams when they work on combinations tasks. They may benefit from moving flexibly between them and in particular using tree diagrams.[9] Explicit teaching of the formula for combinations seems to disturb the development of intuitive empirical strategies for this type of problem.[10] Finding and creating patterns are an important first step and the use of a meaningful context makes it possible for young children to work effectively to find all the possible combinations.[11] Three stages of systematisation have been suggested: *the random stage* (trial and error), *the transitional stage* (some pattern evident) and *the odometer stage* (an organised structure where one thing is held constant and others are changed systematically).[12] *Systematic variation* (changings items in an order), *constancy* (keeping one item the same while you vary others), *exhaustion* (recognising when one constant item has been used in all possible ways) and *completion* (when all constant items have been exhausted, all possible combinations have been found) are the four important principles of working systematically that must be understood to be able to solve combinatorial tasks efficiently.[12]

IMPLICATIONS: It is suggested that students use a variety of representations (pictographic, iconic and symbolic) including sketches, lists, tables and especially tree diagrams when solving combinations problems

The use of a meaningful/real world context allows students to solve combinations problems more effectively

Students who use pattern and structure when solving combinations problems move more easily towards working systematically

4

Children may find combinatorial counting difficult because it involves the key process of moving away from their experience in counting with natural numbers and moving towards beginning to count with a pair of units (or more). They must identify that this is still counting, and monitor their counting to decide when they are going to stop.[13] They may also struggle at first with generalisation (if there are 6 ways of choosing 3 items, how many for other numbers of items?), requiring multiple opportunities to practise with simple examples before moving on to exploring more dynamic problems (considering the general case and representing relationships).[14]

IMPLICATIONS: Early focus on counting units is likely to allow young children to enumerate more flexibly, supporting early work in combinations

Working with combinations may allow students multiple opportunities to generalise, working from simple problems to those that begin to explore modelling relationships

"We usually think of counting as a simple process. However, it can become complicated if whatever we wish to count cannot be readily visualized. In combinatorics, we are more concerned with counting ways of carrying out certain procedures rather than actually counting physical objects"

McFaddin, 2006

"Combinatorics comprises a rich structure of significant mathematical principles that underlie several areas of the mathematics curriculum, including counting, computation, and probability"

English, 1993

Lucy Rycroft-Smith, Darren Macey & Nicky Rushton, 2020

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