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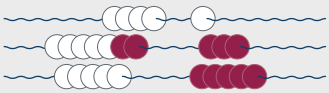
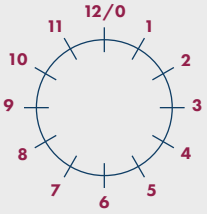
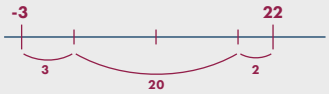
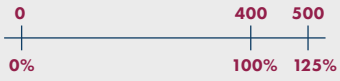
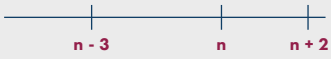
TALKING POINT:

WHAT DOES RESEARCH SUGGEST ABOUT THE NUMBER LINE?

IN SUMMARY

- A number line is a (usually linear) spatial representation of number which helps support mathematical understanding and connections between measure, number and statistics
- Using a number line representation, which is likely to be innate for young children, may form an important part of understanding place value and proportional reasoning
- Number line estimation tasks are suggested as an efficient way to assess broader mathematics competence
- Young children can use bead strings or linear number tracks as early number lines, later moving to notched number lines
- Use of an empty number line model is suggested as it is flexible, allows for student methods/errors to be easily made visible, and is supportive of a wide range of methods
- Training students to use and construct number line models based on their "innate" mental number lines supports mathematical development, and the use of the empty number line can promote conceptual understanding
- Double number lines can help support proportional reasoning, especially highlighting the idea of multiplication as scaling

The number line: a flexible and useful model

Bead string / abacus	
Number track	
Structured number line	
Wrapped number line	
Empty number line (with examples added)	
Double number line	
Variable number line	

1

The idea of *the number line* may refer to various different models of placing numbers on (or reading them from) a line, length of track, scale, or string. It is a linear spatial representation of number that is considered to be an important model in supporting and developing connections between work in measure, number and statistics – in particular sorting quantity and data into space on the Cartesian plane[2,3] – as well as moving from arithmetic to algebraic structure.[4] Using the number line successfully is associated with a developing understanding of place value and in particular proportion,[5] because it is a representation that allows reasoning about continuous quantities.[6] Use of the number line throughout a student's mathematical career can help develop their conceptions of the density of number from counting numbers, to integers, to rational numbers and irrational numbers.[7]

IMPLICATIONS: The number line is a model that helps support mathematical understanding and connections between measure, number and statistics

Using a number line helps support development of place value and proportional reasoning

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Research suggests that children possess an innate sense of a number line whose orientation (whether left-right or right-left for smaller-larger numbers) may be related to the direction of reading they have learned.[8] Researchers currently do not agree whether this early mental number line is linear (equally spaced) or logarithmic (the numbers get closer together as they get larger) or a piecemeal composite structure.[9] Performance on a simple number line estimation task has been found to correlate strongly with performance on counting, arithmetic, and standardised school achievement tests, and can therefore be used as a good proxy for measuring broader mathematical competences which do not need prior knowledge.[10] Using a number line model successfully has been described as making use of this innate mental structure to develop it into a tool to reason with; a shift from just a *model of* to also a *model for*.[11]

IMPLICATIONS: Children seem to have a mental conception of a number line structure that is likely to be left-right smallest-largest if they have learned to read left to right, and vice versa

Number line estimation tasks are suggested as an efficient way to assess broader mathematics competence

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Young children can use coloured beads on a string or other abacus-type structures, emphasising fives and tens, to support spatial representations of number.[12] Moving from this kind of representation to that of a number line (points/notches labelled) is challenging[13] and should be developed in parallel with developing ideas about pacing off length. Later, working on a structured line (some marked numbers provided) may give way to use of the empty number line (no numbers or markers; students add these). The empty number line is recommended because it is a model for number operations that easily allows methods (and errors)[14] to be visible. As a model, it is open to informal strategies while supporting the development of more formal and efficient strategies; it also enhances the flexibility of mental strategies,[15] and may be important in allowing student autonomy as it can be used to record strategies rather than prescribing them.[16]

IMPLICATIONS: Young children's work on spatial representation of number can be supported with structured bead strings or linear number tracks

Moving students from number tracks to number lines (notches labelled rather than spaces) is important but may be challenging for them

Use of an empty number line model is suggested because it allows student methods and errors to be easily made visible, is flexible, and supports student autonomy in choice of methods

4

There is evidence that training students to use the number line more competently may support mathematical development, especially in arithmetic and in particular through the use of number tracks in games (but not when the number tracks are circular, only when linear).[17] Construction of a linear number line allowed students at age 6 to solve a wide range of maths problems more successfully.[18] The use of the double number line helps extend the metaphor of multiplication as stretching, as well as helping students understand that quantities continue to extend beyond the "whole" unit identified (e.g. it can be meaningful to find 125%).[19] Using empty number lines, particularly as an additive reasoning tool at first,[20] can promote conceptual understanding and reduce reliance on counting strategies, but teacher competence is an important mediating factor[21] and students need clear, explicit teaching on how to use them.[22]

IMPLICATIONS: Training students to use and construct number line models supports mathematical development; linear number tracks in games are better for this than circular ones

Double number lines can help support proportional reasoning, especially highlighting the idea of multiplication as scaling

The use of the empty number line, in particular for additive reasoning, can promote conceptual understanding, provided teachers support students in using it

"The number line—a conceptual tool that allows for numbers to be conceived as locations along a line mapping numerical differences onto differences in spatial extension… is mathematically simple, yet it is extraordinarily powerful"

Núñez, 2017[1]

"The empty number line [is] a very powerful tool for enhancing communication in the classroom"

Bobis, 2007[16]

Lucy Rycroft-Smith & Tabitha Gould, 2021

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