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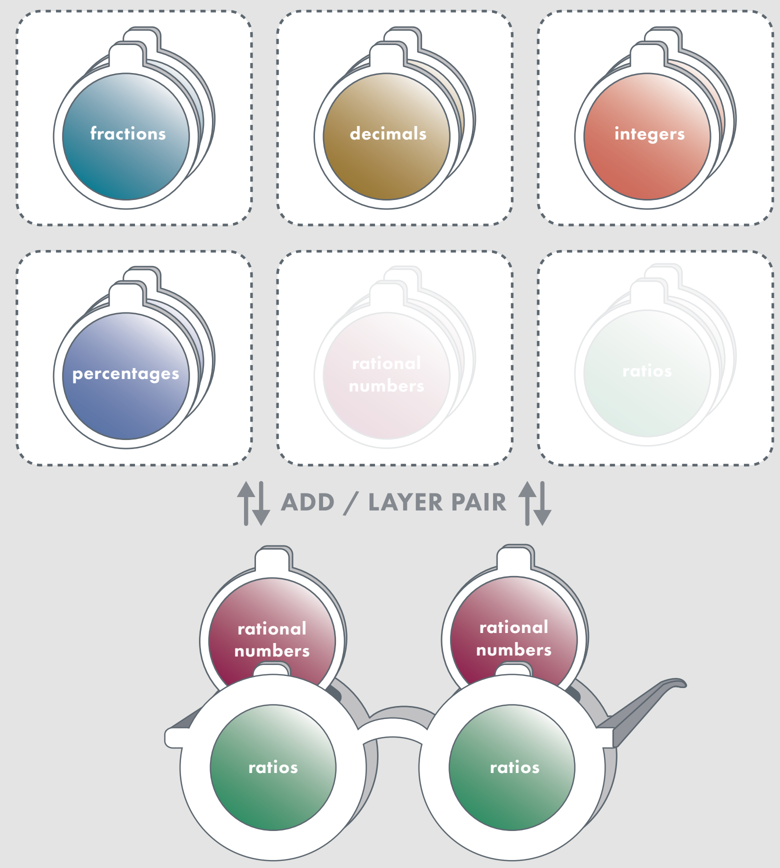
TALKING POINT:

WHAT DOES RESEARCH SUGGEST ABOUT DEVELOPING CONCEPTS OF RATIO?

IN SUMMARY

- Ratio is a well-researched but complex representation which may be best considered as overlapping with other representations of proportion, such as fractions, and is linked to multiplicative thinking, equivalence, scaling and similarity
- Ratio can be used and interpreted in different ways, including part-whole, whole-whole or part-part relations
- Providing opportunities for students to explore the difference between using natural and rational numbers supports teaching and learning about ratio
- Careful variation in contexts of ratio tasks, moving through stages of multiplicative thinking and using technology, supports the development of student understanding of ratios
- Considering possible misconceptions, often arising out of applying additive strategies, can support teachers in effective teaching of ratio
- Some strategies may support proportional reasoning more strongly than others; for example, the "unit value" method
- Double number lines, ratio tables and other models may be useful to teachers and students

Overlapping lenses of proportional reasoning



Adapted from Clark, Berenson and Cavey, (2003, p. 308).[6] Used with permission from the publisher, Elsevier

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Ratios are one form of representation of mathematical relations between quantities, and part of the wider ideas of proportion and rational number reasoning.[3] Ratios are representations that imply a multiplicative relationship, suggesting a calculation of multiplying or dividing parts by one another, and connected to ideas of scaling, equivalence and similarity.[4] Teaching ratio – inextricably linked to, and part of, proportional reasoning – is complex[5] both to teach and to learn.[4] Proportional relationships as expressed in fractions, decimals, percentages, and ratios have been debated in "enormous amounts of" research,[5] and various related choices made in curricula and teacher support materials. In one study, it was found that the majority of maths teachers saw fractions as a subset of ratios, whereas seeing the two as overlapping sets instead is suggested as beneficial.[6]

IMPLICATIONS: Ratios are complex representations of rational numbers that are linked to multiplicative thinking, scaling and congruency

There is a great deal of research considering teaching and learning with ratios, both of which are challenging

Perceiving fractions and ratios as overlapping sets, rather than fractions as a subset of ratios, may be useful for teachers

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Ratios are linked to other representations of multiplicative relations and comparisons between quantities, such as fractions, decimals, percentages, and other visualisations of proportion such as slope. Considering ratio alongside these as overlapping lenses of proportional reasoning can be helpful (see infographic).[6] In particular, the distinction between ratio and fractions may be hard to define.[6] This is related to both multiple interpretations and uses of ratios as part-whole, whole-whole and part-part relations,[4] as well as the multiple types of reasoning with ratios including maintaining equivalence, comparing ratios, visualising dynamic relationships, and changing proportions of a static whole.[7]

IMPLICATIONS: Considering ratio as a lens which overlaps with other representations of proportion may support teachers in teaching ratio

It may be useful for teachers to attend to the use and interpretation of ratios in different ways, emphasising their possible ambiguity when it comes to part-whole or part-part relations

3

Student understanding of ratio is related to moving from natural number to rational number reasoning. This movement may be supported by the understanding that there are many natural number concepts, such as ordering, use of symbols and inverses, that cannot be simply transferred to rational numbers.[3] In one study, systematically varying the contexts in which ratios occur, according to a particular method of classification, and supporting this with the use of technology tools, was associated with higher attainment than a control group which did neither.[5] Other suggestions for useful ways to teach ratio include a multiplicative reasoning "learning path" – from seeing multiplication as repeated addition through patterns in the multiplication table to seeing rates as static (columns in the multiplication table) and then, more dynamically, through the use of ratio tables.[8]

IMPLICATIONS: Teaching and learning ideas of ratio is supported by allowing students to grapple with the difficulties of extending/refining ideas of natural numbers to rational numbers

Careful variation in contexts of ratio tasks and the use of technology may support more successful proportional reasoning from students

It is suggested that moving students through foundational ideas of multiplication and multiplicative reasoning to ratio tables may help them be more successful when learning about ratios

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Student understanding of ratio is linked not just to reasoning proportionally, but specifically to recognising the difference between additive and multiplicative thinking. Student errors and misconceptions can be the starting point for the effective teaching of this topic; looking, for example, at using incorrect additive/constant difference strategies (adding/subtracting the same amount to all parts of the ratio), constant sum strategies (aiming to maintain the total sums of two "equivalent" ratios rather than the proportions of the parts), and "magical doubling" strategies (just doubling without understanding).[9] The "build-up" strategy (e.g. 300 g sugar for 4 people, 600 g sugar for 8 people, another 2 people is half of 4 people, so that we need 750 g sugar for 10 people), very often used by pupils, has been criticised as "a relatively weak indicator of proportional reasoning."[10] One suggested strategy to make pupils aware of is finding a unit value that can be scaled up or down, also referred to as the "little recipe" or "basic combination."[7] The use of models such as pictures, ratio tables and double number lines is also recommended to support proportional reasoning.[11]

IMPLICATIONS: Considering possible misconceptions, such as applying additive or blindly procedural strategies, can support teachers in effective teaching of ratios

Some strategies may support proportional reasoning more strongly than others; for example, the unit value scaling method

The use of models such as images, double number lines and ratio tables is recommended

"Mathematics educators often talk right past one another because they are using *fraction*, *ratio*, or *proportion* in different ways without realizing it"

Smith, 2002[1]

"All of the following topics involve ratio in one way or another: changing between different units of measurement; time and speed; exchange rates; recipes; graphs of straight lines through the origin. Thus, many mathematical topics are essentially contexts for dealing with ratio questions"

Griffin, 1988[2]

Lucy Rycroft-Smith & Tabitha Gould, 2021

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