ISSUE 37 JUNE 2021

CAMBRIDGE Mathematics

ESPRESSO

RESEARCH, FILTERED BY CAMBRIDGE MATHEMATICS

TALKING POINT:

WHAT DOES RESEARCH SUGGEST ABOUT TEACHING AND LEARNING EARLY CONCEPTS OF PROBABILITY?

IN SUMMARY

- It is important for students to learn using both theoretical and empirical aspects of probability, reasoning about short- and long-term behaviour
- Probabilistic reasoning depends on proportional reasoning, especially the flexible use of fractions and estimation
- Development of the important concept known as the law of large numbers is important; for example, students making and analysing short-, medium- and long-term predictions about probability situations
- Helping students get a sense of randomness in a variety of different task types, including categorising and creating random sequences, is suggested
- Tasks which focus on the concept of equalising or equivalence are also recommended as useful, as is the use of carefully designed technology tools
- It is useful for teachers to be aware of common biases, ambiguities in language and misconceptions around probability

Common forms of bias when people think about probability

AVAILABILITY BIAS

A tendency to assign a greater likelihood to things that are easy to remember

"It always rains on a Bank Holiday"

REPRESENTATIVENESS BIAS

A tendency to assign likelihood of an outcome based on it appearing similiar to other known outcomes

"I didn't need one yesterday so I won't need an umbrella today"

EQUIPROBABILITY BIAS

A tendency to assume different outcomes are equally likely

"It can either rain or not rain on any day, it's 50% either way"

Adapted from ideas in Pratt, D. (2000).[1]

1

Learning about probability involves looking for patterns – recognising that stability and regularity develop with frequency – as well as assigning and comparing numerical probabilities, and applying understanding of proportion to situations involving uncertainty, risk, likelihood and prediction.[4] An understanding of randomness in relationships underlies statistical reasoning and the scientific method in general.[5] In particular, students should experience, at first in informal ways, both the *theoretical* (calculating with the sample space) and *empirical* (collecting and analysing data from experiments) aspects of probability, which are complementary.[6] Theoretical approaches alone may only provide artificial and simplified examples, removed from real-world decision making.[7] Exploring probability experiments helps support the idea that theoretical probabilities are a model of what to expect that is only valid in terms of long-term behaviour.[6]

IMPLICATIONS: Reasoning with probability and randomness is extremely important as it underlies all statistical activity and scientific thinking

It is important for students to learn using both theoretical and empirical aspects of probabilistic reasoning through early, informal experiences; this helps connect real-world decision making to calculation and reasoning about short- and long-term behaviour

"We regard experimentation as vital to understanding the role of chance and unpredictability"

Gage & Spiegelhalter, 2016[2]

"A poll asked people what 'serious possibility' meant, [obtaining] figures from 20% to 80%. Such a wide range was clearly a problem, as the policy implications of those extremes were markedly different"

Mauboussin & Mauboussin, 2018[3]

CAMBRIDGE UNIVERSITY PRESS | UNIVERSITY OF CAMBRIDGE Faculty of Mathematics | UNIVERSITY OF CAMBRIDGE Faculty of Education | Cambridge Assessment

2

The distinction between short- and long-run effects in probabilistic situations is not always clear to students.[8] This idea is underpinned by the important concept of the *law of large numbers* (probabilities stabilise towards their theoretical counterpart as the number of trials increases). This can be used to link theoretical and empirical perspectives of probability by drawing attention to the way that these converge as the number of trials increases, and recognising or creating situations in which to explore equivalence in terms of approximate proportions.[6] Focusing explicitly on whether predictions are successful in the short, medium and long term can help highlight important misconceptions such as issues with the strategy of betting on a favourite number.[9]

IMPLICATIONS: Helping students to see that theoretical and empirical probabilities converge with repeated trials, in particular exploring them as potentially equivalent proportions, supports development of the important concept known as the law of large numbers

Students making short-, medium- and long-term predictions about probability situations and evaluating their strategies supports working through misconceptions and understanding the law of large numbers

3

Students' understanding of probability benefits from being able to move between numerical and visual representations of fractions, decimals and percentages, and gaining familiarity with comparisons of magnitude expressed in different ways (for example, 1 in 200, or 50% higher risk).[10] Another important foundational concept is the hard-to-define idea of randomness: "The understanding of the implications of randomness lies at the centre of all statistical thinking" (p. 3).[5] Students should experience several types of task to develop a sense of randomness. These include exploring *random mixtures* (e.g. repeated shuffling of a brand new pack of cards); *random distributions* (e.g. students drawing a random pattern of raindrops on a grid to mimic rainfall on a paving slab); and *random draws/random generators* (e.g. games with dice, spinners or beads in a jar).[4] Activities in which students aim to equalise probabilities by changing proportions – for example adding extra beads to one jar – can support the important connection between proportional and probabilistic reasoning.[11] Software tools can support a useful approach to reasoning with probability in which real-world phenomena are modelled and simulated.[12]

IMPLICATIONS: Students' understanding of probability benefits from foundational work in proportional reasoning and comparing magnitudes with fractions, decimals and percentages

Helping students get a sense of randomness by encouraging classroom experiences with random mechanisms and processes in a variety of different task types is suggested

Tasks which focus on the concept of equalising or equivalence are also recommended as useful, such as adding, subtracting or swapping beads in jars and considering the effect on the ensuing proportions

Using software tools can support students in using and exploring probabilities as part of sensemaking

4

There are common biases to address in teaching and learning probability; for example, *availability bias* (treating things that are easy to recall as more likely – see infographic).[1] Another issue in developing probability literacy is the need for a nuanced recognition of the ambiguities of different types of language used around probability; for example, recognising that the word "likely" has a wide range of possible meanings.[13] Students have difficulty with recognising randomness in sequences, tending to expect too many short runs and discount longer runs (e.g. for repeated coin flips); tasks in which students categorise sequences as random or not random, or attempt to create convincing random-looking sequences, may help highlight these misconceptions.[14] Students also have trouble understanding independence of events; for example, that each outcome of a coin flip has no bearing on a previous or successive one (known as *recency error*).[5]

IMPLICATIONS: It is useful for teachers to be aware of common biases and misconceptions around probability; for example, that students tend to treat things that are easier to recall as more likely and may struggle to understand independence of events such as coin tosses

Exploring ambiguities around the ways that probabilities can be expressed can support reasoning about communication in the context of risk and uncertainty

Students often struggle to understand randomness; asking them to categorise sequences of outcomes as random or not, or to attempt to create their own, may help

Lucy Rycroft-Smith & Darren Macey, 2021

REFERENCES

1. Pratt, D. (2000). Making sense of the total of two dice. *Journal for Research in Mathematics Education, 31*(5), 602–625.
2. Gage, J., & Spiegelhalter, D. (2016). *Teaching probability*. Cambridge University Press.
3. Mauboussin, A., & Mauboussin, M. J. (2018, July). If you say something is "likely," how likely do people think it is? *Harvard Business Review.*
4. Langrall, C. W., & Mooney, E. S. (2005). Characteristics of elementary school students' probabilistic reasoning. In G. A. Jones (Ed.), *Exploring probability in school: Challenges for teaching and learning* (pp. 95–119). Springer.
5. Bryant, P & Nunes, T. (2012). *Children's understanding of probability: A literature review* [Full report]. Nuffield Foundation.
6. Prodromou, T. (2012). Connecting experimental probability and theoretical probability. *ZDM, 44*(7), 855–868.
7. Helmerich, M. (2015, February). *Rolling the dice – exploring different approaches to probability with primary school students* [Paper presentation]. CERME 9 – Ninth Congress of the European Society for Research in Mathematics Education, Prague, Czech Republic. (pp. 678–684).
8. Konold, C. (1989). Informal conceptions of probability. *Cognition and Instruction, 6*, 59–98.
9. Schnell, S., & Prediger, S. (2012). From "everything changes" to "for high numbers, it changes just a bit": Theoretical notions for a microanalysis of conceptual change processes in stochastic contexts. *ZDM, 44*(7), 825–840.
10. Gal, I. (2005). Towards "probability literacy" for all citizens: Building blocks and instructional dilemmas. In G. A. Jones (Ed.), *Exploring probability in school: challenges for teaching and learning* (pp. 39–64). Springer.
11. Falk, R., & Wilkening, F. (1998). Children's construction of fair chances: Adjusting probabilities. *Developmental Psychology, 34*(6), 1340–1357.
12. Batanero, C., Chernoff, E. J., Engel, J., Lee, H. S., & Sánchez, E. (2016). *Research on teaching and learning probability*. Springer.
13. Mosteller, F., & Youtz, C. (1990). Quantifying probabilistic expressions. *Statistical Science, 5*(1), 2–12.
14. Falk, R., & Konold, C. (1997). Making sense of randomness: Implicit encoding as a basis for judgement. *Psychological Review, 104*(2), 301–318.

CAMBRIDGE UNIVERSITY PRESS
UNIVERSITY OF CAMBRIDGE Faculty of Mathematics
UNIVERSITY OF CAMBRIDGE Faculty of Education
Cambridge Assessment