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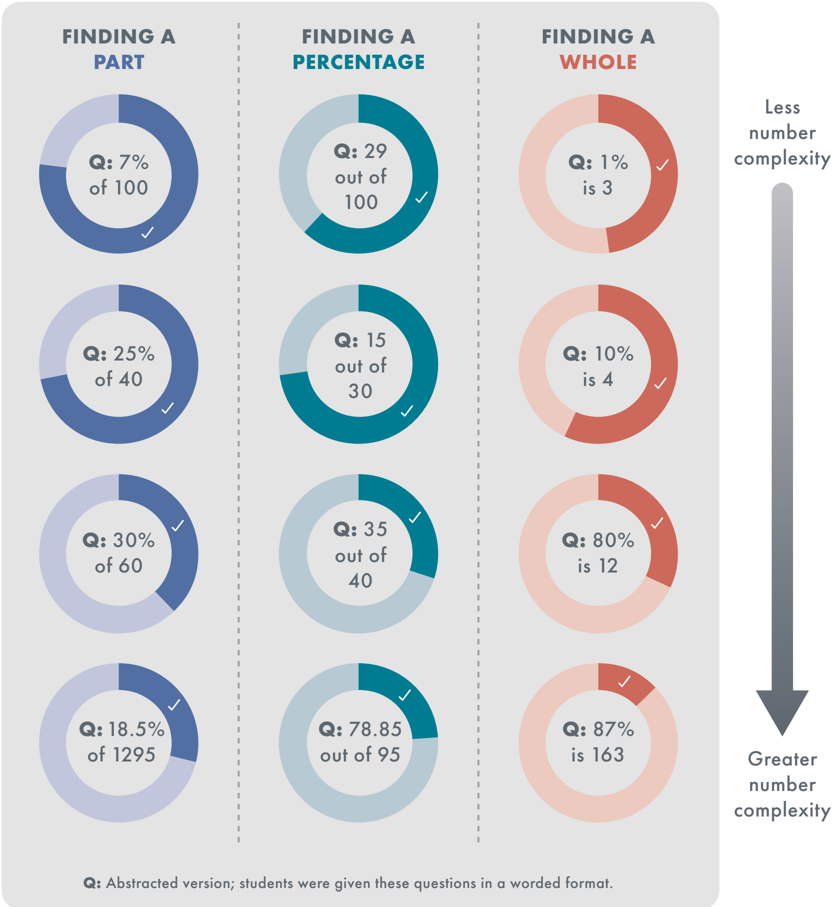
TALKING POINT:

WHAT DOES RESEARCH SUGGEST ABOUT THE TEACHING AND LEARNING OF PERCENTAGE?

IN SUMMARY

- Percentages are commonplace but are an under-researched representation of proportion
- Approaches grounded in proportional reasoning with real-life examples are recommended over those that are procedural or atomised
- A good grounding for early percentage thinking involves comparison, correspondence and considering "so-many-per-so-many" situations, working on fractions first so that students can visualise percentage as parts-per-hundred
- Students may be asked to find a percentage part of a whole, represent a part of a whole as a percentage, or find the whole when given a percentage part; this last is often the hardest for them
- Students also find dynamic modelling of percentage change difficult as it involves change in what constitutes the "whole"
- It is suggested that early opportunities to develop and explore percentage benchmarks of 1, 5, 10, 25, 50 and 100 are beneficial
- Students should have opportunities to move flexibly between representations such as the bar, ratio table, double number line, and their own representations of percentage

Proportion of correct responses (✓) by Year 9 students to different types of percentage question (calculator provided)



Data from Baratta et al. (2010)[2]

1

Percentages are used extensively in the real world and across the curriculum.[1] Percentage is integrated into the broader concept of proportion and proportional reasoning, the challenges of which have been widely reported[2] and learning should be embedded within these wider ideas.[3] It is suggested that one reason for students struggling to interpret and apply percentage concepts in different contexts is an emphasis on procedures and recall, rather than deep understanding.[4] There is a lack of research into teaching and learning percentage and wide disparity among suggested approaches.[5]

IMPLICATIONS: Percentages are likely to be encountered outside the mathematics classroom, both in other subjects and in the real world

Percentages are an important and commonplace representation of proportion, but are currently under-researched; learning about percentage should be embedded within proportional reasoning

If students are only given procedural or atomised teaching around percentages, they are likely to have less success than with approaches grounded in reasoning with real-life examples

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As with proportion more generally, early percentage (development of thinking about comparison and correspondence) can develop out of "so-many-per-so-many" situations.[4] Connections with developing fraction concepts are considered important.[6] An emphasis on visualising percentages as parts-per-hundred is suggested,[2] and so familiarity with hundredths in fraction notation as well as some sense of equivalent fractions should precede more formal teaching about percentages.[6] Two approaches to introducing percentages are suggested: the first simply presents percentages to students as an alternative notation to represent part-whole relationships; whereas the second is rooted in comparison situations with fractions, where a common reference of 100 can emerge as useful to students after more critical thinking.[6] Percentage use also arises from students themselves as a way of reasoning with proportional comparison in data handling contexts.[7]

IMPLICATIONS: Early percentage thinking involves comparison, correspondence and considering "so-many-per-so-many" situations

Research suggests that where students have opportunities to develop some fraction knowledge before considering percentage, they will be more able to relate percentages explicitly to fraction concepts by visualising them as parts-per-hundred

Percentages can be presented to students as a particular form of proportion notation, or can emerge from more critical thinking involving the need to compare amounts across a common reference

3

Students are first likely to encounter percentage problems that fit into three categories: finding a percentage part of a whole, representing a part of a whole as a percentage, and finding the whole when given a percentage part.[5] Students reportedly find the third type of problem – finding the whole when given a percentage part – more difficult, especially when using decimal notation, as it often involves division by a number less than 1 (see infographic).[2] More complex problems involving percentage change, multiple percentages or percentages that are compounded are considered the most challenging for students. This is due to the difficulty of identifying the "whole" relevant to a particular part of the problem[8] and that these problems require students to conceive of percentages as operators (in various representations); that is, "that adding a percent is the same as multiplying."[2(p68)]

IMPLICATIONS: Three broad types of initial percentage problems have been identified: finding a percentage part of a whole; representing a part of a whole as a percentage; and finding the whole when given a percentage part, of which this last is the hardest for students

Students also find dynamic modelling of percentage change difficult as it involves reconceptualising or paying particular attention to change in what constitutes the "whole"

4

Research recommends explicitly relating early development of benchmark or "anchor" percentages such as 1, 5, 10, 25, 50 and 100 percent to corresponding proportions in fraction and decimal notation. These, combined with representations that include the bar, double number line and ratio table, can support flexible mental arithmetic with percentages.[6] The ratio table is considered a helpful representation, offering the freedom to make different interim steps, but is deemed most powerful when used in combination with the bar or double number line because it helps students move between representations and see structural differences and similarities.[3] Students aged 10–11 years who were guided towards designing their own representations of percentages were more successful than those who were given ready-made representations.[9]

IMPLICATIONS: Developing early benchmark percentage "anchors" of 1, 5, 10, 25, 50 and 100 with students and relating them to fraction and decimal notation is recommended

Flexibly moving between representations such as the bar, ratio table and double number line, and guiding students to construct their own representations of percentage supports students' successful reasoning strategies

> "Journalists and percentages mix like ball bearings in soufflé"
>
> **Blastland (2008)[10]**

> "One must not forget that a student's understanding of percentages has its non-numerical, contextual roots"
>
> **van den Heuvel-Panhuizen et al. (1995)[11(p25)]**

Tabitha Gould, Lucy Rycroft-Smith & Fran Watson, 2021

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