ISSUE 40 DECEMBER 2021

CAMBRIDGE Mathematics

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RESEARCH, FILTERED BY CAMBRIDGE MATHEMATICS

TALKING POINT:

WHAT DOES RESEARCH SUGGEST ABOUT THE TEACHING AND LEARNING OF FRACTION EQUIVALENCE?

IN SUMMARY

- Students need equivalent fractions to understand fractions as quantities, but this can be challenging because "equivalent fraction" can mean the same label for different amounts or a different label for the same amounts
- Students should not just multiply and divide equivalent fractions, but use different area, collection and number line models, flexibly composing, decomposing, partitioning and repartitioning them
- It is recommended that students are provided with opportunities to represent equivalent fractions in different ways, such as pictures, pie charts, ratio tables, fraction bars and double number lines, and in particular to create their own representations
- Focusing on partitioning with students in early work on fraction equivalence may cause difficulties; a correspondences approach is suggested instead
- Modelling decimal equivalence can help students to see that renaming a quantity does not change its size
- To develop fraction equivalence, students should have opportunities to explore constructing, manipulating and mentally repartitioning diagrams, including those where the parts are not equal, and using diagrams which represent examples such as ½ = ½ as different shapes

Two ways of thinking about division and fractions

PARTITIONING also called subdivision or dissection	**CORRESPONDENCES** also called dealing or sharing
Cutting a whole into a predetermined number of equal parts; for example, "show $\frac{3}{8}$ of a pizza"	Dividing to obtain a single (may be fractured) quantity; for example, "divide 3 baguettes among 5 people" — $\frac{3}{5}$ of a baguette each
Relates to a single whole	Relates to two quantities or measures — *x per y*
Used for part-whole contexts	Used for quotient ("per") contexts
Within-quantity relations (the more parts, the smaller the parts)	Between-quantity relations (the more children, the less of the quantity each)

Adapted from Nunes & Bryant (2009)[3]

1

The concept of equivalent fractions is important for comparing and calculating with fractions, because to understand fractions as numbers (quantities that can be placed on a number line), students must be able to know whether two fractions are equivalent or not, and if they are not, which one is the bigger number.[3] However, equivalent fractions can be challenging for students, because an equivalent fraction can be thought of as "a number that has many names."[4(p427)] For example, two fractions with the same label can be non-equivalent (e.g. ⅓ of 12 is not equivalent to ⅓ of 21) and yet two fractions with different labels can be equivalent (e.g. ¾ of a particular whole is an equivalent quantity to ⅝ of the same whole).[3]

IMPLICATIONS: Students need the concept of equivalent fractions in order to understand fractions as numbers (quantities that can be placed on a number line)

It may be useful for both teachers and students to explore the challenging dual nature of the term "equivalent fraction" (same label for different amounts or different label for same amounts) by considering the whole to which each is referring

> "There is no single path to competence [regarding the understanding of fraction equivalence]. But some paths are travelled more than others"
>
> **Pellegrino et al., (2001)**[1(p182)]

> "I can remember when I was in school my teacher would call simplifying a fraction reducing"
>
> **Williams, (n.d.)**[2]

2

There is more to students' understanding of equivalent fractions than merely multiplying or dividing the numerator and denominator of a fraction by the same number.[5] An integrated understanding is suggested as a combination of: seeing a fraction as a quantity in relation to a referent unit and as a member of an equivalence class; being able to use partitioning area, collection or number line models; repartitioning, constructing or chunking to construct equivalent fractions; and using symbolic notation.[5]

IMPLICATIONS: Students benefit from going beyond just multiplying and dividing the numerator and denominator of a fraction when exploring equivalent fractions

They are supported in developing an integrated understanding of fraction equivalence by using different area, collection and number line models, and flexibly composing, decomposing, partitioning and repartitioning as part of this modelling process

3

Children as young as 5- or 6-years old have been shown to be able to reason about equivalency and the ordering of fractions when using an initial approach which focuses on *correspondences* (or dealing) in a quotient situation, as opposed to starting with *partitioning*[3] (see infographic). Partitioning (sub-dividing a whole to consider equal parts) can be used in reasoning about part-whole tasks, but this can cause difficulties for students in their understanding of fraction equivalence if used as the sole and initial teaching strategy. A correspondences approach has been shown to spotlight the idea that different fractions can represent the same quantity, making it easier to develop an understanding of improper fractions and leading to insight into the inverse relation between the *quotient* (result of division) and the *divisor* (quantity being divided by).[3]

IMPLICATIONS: Two contrasting models for supporting reasoning about equivalent fractions are partitioning (sub-dividing a whole to consider equal parts) and correspondences (dealing or sharing to create a single quantity)

An initial approach limited to partitioning with students in early work on fractions may cause difficulties in seeing equivalence

Using a correspondences approach can support students' understanding of equivalence between differently named fractions, the concept of improper fractions and the inverse relation between the divisor and the quotient

4

Working with a range of models such as pictures, pie charts, ratio tables, fraction bars, and the double number line supports pupils in accessing ideas of equivalence at differing levels of abstraction and formality.[6] Modelling decimal equivalence with base ten manipulatives and a place value chart has been shown to support an understanding of equivalent quantities, including the appreciation that renaming a quantity does not change its properties or size.[7] Encouraging students to create their own representations of equivalent fractions may support a more connected understanding of the concepts involved, compared to solely offering them prepared models and images.[8]

IMPLICATIONS: Working with different models such as pictures, pie charts, ratio tables, fraction bars and the double number line helps students access ideas of fraction equivalence at different levels

Manipulatives and activities that model decimal equivalence can help students to see that renaming a quantity does not change its size

A more connected understanding may result from students creating their own representations of equivalent fractions, rather than receiving prepared templates

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To recognise fraction equivalence when using area models, students need to construct, manipulate and mentally re-partition diagrams, ignoring or imagining partition lines.[e.g.8,9,10] This is particularly challenging when dealing with diagrams that contain both equal and unequal parts, as students may have a strong sense that fractions should be represented by equal shares of a whole.[11] Exploring area models of equivalence should include examples such as ½ = ½ where these are presented as different shapes.[8]

IMPLICATIONS: To develop fraction equivalence, students should have opportunities to explore constructing, manipulating and mentally repartitioning diagrams, including those where the parts are not equal (which may challenge previous ideas about fractions)

Using diagrams with differently shaped partitions to represent examples such as ½ = ½ is suggested

Fran Watson, Lucy Rycroft-Smith & Tabitha Gould, 2021

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