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Espresso

A sip of filtered research

CAMBRIDGE Mathematics

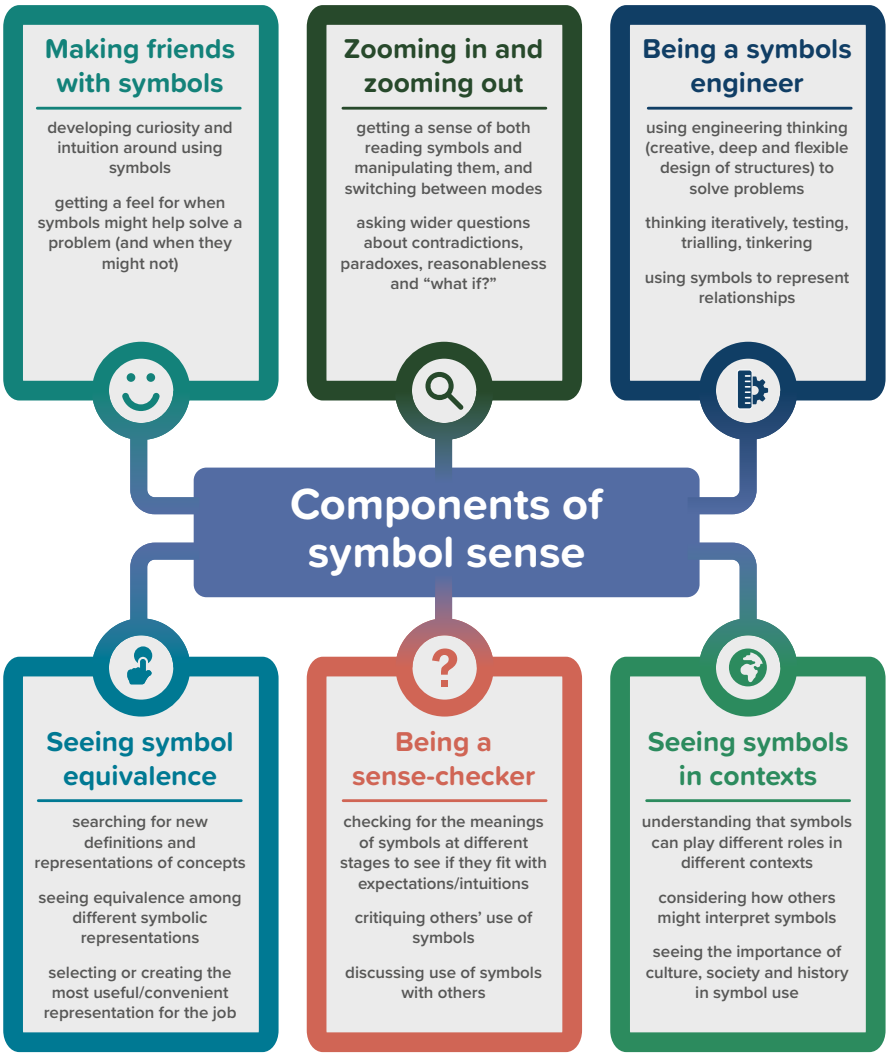
Lucy Rycroft-Smith & Darren Macey, 2022

Talking point

What does research suggest about the development of symbol sense?

In summary

- Symbols are tools or referents to help create meaning and express relationships; for example, numerals, arithmetic operators, and the equal sign, as well as pictorial representations, icons and gestures
- Symbol sense (sometimes called structure sense) refers to using symbols with understanding, including when to use symbols and choosing usefully between them as part of a process of meaning-making
- Using symbol sense is foundational to the development of early algebraic thinking, in particular moving from a local focus to a global, more whole-centred viewpoint
- Symbol sense may be supported by students exploring the purpose and consequences of creating their own representations or graphics from an early age
- Carefully chosen ICT tools can help support the development of symbol sense, as can an attitude of patience towards incomplete understanding and uncertainty
- Exploring quantity in terms of equivalence and balancing nested structures such as mobiles may also support symbol sense development
- Successful manipulation of symbolic expressions does not necessarily imply symbol sense; classroom culture should prioritise conceptual as well as procedural understanding in order to develop symbol sense



Adapted from ideas in Arcavi (1994, 2005).[1,2] © FLM Publishing; used with permission.

A symbol is simply something that can be used to refer to an object because the object itself is too large or small, far away, or not physical at all such as an idea or an abstraction.[3] Unlike notation, which may refer to shorthand or abbreviation, symbols provide an association to help perceive, recognise or create meaning.[4] It has been suggested that symbols are tools for understanding the world – only useful when they sum up in shorthand something which has already been experienced.[5] Symbols may include, for example, numerals, arithmetic operators, and the equal sign, as well as pictorial representations, icons and gestures. *Symbol sense* is a recent term in maths education, similar to number sense, which is usually defined as an appreciation for symbols and when to use them[6] as well as developing an intuition for flexibly understanding equivalence, context and levels of zoom when expressing ideas symbolically (see infographic).[1,2] *Structure sense* is sometimes used in a similar way to symbol sense; for example, defined by one researcher as "an intuitive ability towards symbolic expressions, including skills to perceive, to interpret, and to manipulate symbols in different roles."[7(p147)]

Implications:

Symbols are tools or referents to help create meaning and express relationships; for example, numerals, arithmetic operators, and the equal sign, as well as pictorial representations, icons and gestures

Symbol sense (sometimes called structure sense) refers to using symbols with understanding, including when to use symbols, appreciating which symbols are equivalent, and choosing usefully between symbols

In order to engage symbol sense, symbols should be part of a process of meaning-making

2 Symbol sense is rooted in students' development from basic calculation skills with a local focus (situated in the specific problem) to more strategic, systematic work with a global focus (considering classes of problems and an emphasis on algebraic reasoning)".[8] Of particular importance is developing a *gestalt view* (seeing a structured whole that is more than the sum of its parts): that is, recognizing patterns, recognizing signals and symbols, and making strategic decisions about what to do next,[9] which has been applied to algebraic expressions but also applies more broadly to working with other symbols and representations. Symbol sense plays an important role in the development of early algebraic thinking, possibly as a mediator between number sense and algebraic thinking along with pattern sense.[10] Although some researchers limit definitions of symbol sense to the development of algebraic reasoning, others have emphasised the importance of allowing students to create their own mathematical graphics as a bridging function between early informal marks and the standard abstract symbols of mathematics.[11]

Implications:

Symbol sense allows students to see "the whole" when using symbols

Using symbol sense is fundamental to development of algebraic thinking, in particular moving from a local view to a global, more systematic focus

Symbol sense may be supported by students exploring the purpose and consequences of creating their own representations or graphics

3 It is suggested that the use of ICT tools that are stable, easy to use for students, mathematically sound and faithful to the underlying mathematical properties can increase both algebraic skill performance and symbol sense in the classroom.[9] Another suggestion is that a particular mathematical attitude supports symbol sense development: one of "intellectual patience towards partial understandings [and] confidence that further actions (not totally clear at the beginning) will advance you."[2(p45)] Exploring quantities in terms of how they relate to one another through equivalence and non-equivalence,[12] and using tasks designed around maintaining the balance of hanging nested structures represented in diagrams,[13] have been proposed as useful ways to support the development of symbol sense.

Implications:

Carefully chosen ICT tools can help support the development of symbol sense

An attitude of patience towards incomplete understanding in mathematics may help to support symbol sense development

Exploring quantity in terms of equivalence and the use of hanging mobiles may also support symbol sense development

4 There is a tension between the type of playing with mathematics for sense-making that helps to develop symbol sense and the culture of classrooms or assessments that rewards only formal "correct answers."[2] Student behaviour that may indicate symbol sense has been measured; for example, in one study students used methods "with insight" rather than "ad hoc strategies," but this relied on researcher definitions of "correct" procedure choice and was limited to algebra.[14] The ability to manipulate mathematical expressions or perform arithmetic and algebraic computations does not necessarily imply an understanding of mathematical meanings, the recognition of structures, or the ability to interpret the results.[15] More research is needed on symbol sense.

Implications:

Successful manipulation of symbolic expressions does not necessarily imply symbol sense

Classroom culture should emphasise and reward activities related to sense-making in order to develop students' symbol sense

> "A symbol which has not been properly defined is not a symbol at all. It is merely a blot of ink on paper"
>
> Whitehead, 1911/2017[16(p52)]

> "From chalk to software code, mathematicians and scientists use a variety of methods to express equations and formulas, and they have different ideas about the meaning behind their numerical prose"
>
> Clavin, 2022[17]

References

1. Arcavi, A. (1994). Symbol sense: Informal sense-making in formal mathematics. *For the Learning of Mathematics, 14*(3), 24–35.
2. Arcavi, A. (2005). Developing and learning symbol sense in mathematics. *For the Learning of Mathematics, 25*(2), 42–48.
3. Pimm, D. (1995). *Symbols and meanings in school mathematics*. Taylor & Francis Group.
4. Mazur, J. (2014). *Enlightening symbols: A short history of mathematical notation and its hidden powers*. Princeton University Press.
5. Dewey, J. (1902). *The child and the curriculum*. University of Chicago Press.
6. Bergsten, C. (1999). From sense to symbol sense. In I. Schwank (Ed.), *European research in mathematics education* (Vol. 2, pp. 126–137). Forschungsinstitut für Mathematikdidaktik; ERME.
7. Jupri, A., Sispiyati, R., & Chin, K. E. (2021). An investigation of students' algebraic proficiency from a structure sense perspective. *Journal on Mathematics Education, 12*(1), 147–158.
8. Drijvers, P., Goddijn, A., & Kindt, M. (2010). Algebra education: Exploring topics and themes. In P. Drijvers (Ed.), *Secondary algebra education: Revisiting topics and themes and exploring the unknown* (pp. 5–26). Sense.
9. Bokhove, C. (2011). *Use of ICT for acquiring, practicing and assessing algebraic expertise* [Doctoral dissertation, Utrecht University]. ResearchGate.
10. Somasundram, P., Akmar, S. N., & Eu, L. K. (2019). Year five pupils' number sense and algebraic thinking: The mediating role of symbol and pattern sense. *The New Educational Review, 55*(1), 100–111.
11. Carruthers, E. & Worthington, M. (2011). *Understanding children's mathematical graphics: Beginnings in play*. Open University Press; McGraw-Hill.
12. Coles, A. (2021). Commentary on a special issue: Davydov's approach in the XXI century. *Educational Studies in Mathematics, 106*(3), 471–478.
13. Papadopoulos, I. (2018). Using mobile puzzles to exhibit certain algebraic habits of mind and demonstrate symbol-sense in primary school students. *The Journal of Mathematical Behavior, 53*, 210–227.
14. Turşucu, S., Pandaw, J., & de Vries, M. J. (2020). Search for symbol sense behavior: Students in upper secondary education solving algebraic physics problems. *Research in Science Education*. Springer. (Original work published 2018)
15. Goldin, G., & Shteingold, N. (2001). Systems of representations and the development of mathematical concepts. In F. R. Curcio & A. A. Cuoco (Eds.), *The roles of representation in school mathematics* (pp. 1–23). National Council of Teachers of Mathematics.
16. Whitehead, A.N. (2017). *An introduction to mathematics*. Dover Publications Inc. (Original work published 1911)
17. Clavin, W. (2002, Summer). *Writing in the language of math*. Caltech magazine.

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