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Espresso

A sip of filtered research

CAMBRIDGE Mathematics

Tabitha Gould, Frances Watson & Lucy Rycroft-Smith, 2023

Talking point

What does research suggest about developing concepts of pattern?

In summary

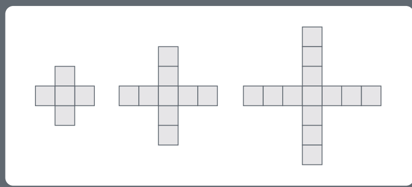
- Pattern and patterning activities are pervasive in the world around us and are central ideas within mathematics
- In school mathematics, children are likely to be introduced to three broad types of pattern: repeating patterns; spatial structural patterns; and growing patterns
- Early patterning activities may have a positive effect on later mathematical attainment, especially representing or duplicating a pattern using different materials from the original (a process called abstraction)
- Pattern activities have the potential to support the development of symbol and structure sense, generalisation, and the use of variables; focusing on describing what stays the same and what changes between pairs of elements in a pattern is encouraged
- Playing with visual patterns that can be decomposed into recognisable parts supports the uncovering of generalised rules

Different ways of seeing the same pattern sequence

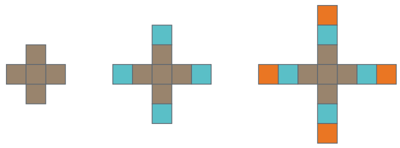
What do you see here?

What's the same and what's changing?

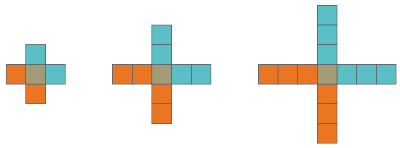
How could it be seen differently?



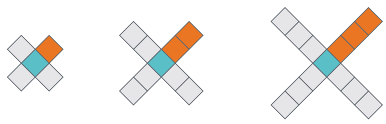
I see adding fours to the first one, to make each new shape



I see [ ] and [ ] shapes shrinking, and they overlap



I see windmills. The centre stays the same and the arms/sails change length



1 Pattern is pervasive in the world around us, and even before formal schooling, young children frequently interact with and are curious about finding pattern in, for example, toys, games, books, puzzles, music and TV shows.[1] Within mathematics, *pattern*, "any predictable regularity, usually involving numerical, spatial or logical relationships",[2(p34)] and *patterning*, "the search for mathematical regularities and structures",[3(p190)] are central ideas that extend well beyond any single content area.[e.g. 1, 4, 5] The teaching and learning of pattern tends to fall within three contexts: repeating patterns – those that have a recognisable recurring cycle of elements;[6] spatial structural patterns, including subitising, geometrical shapes, arrays, measurement units and data organisation;[7] and growing patterns, where elements follow a certain sequence that is increasing or decreasing in dimension or quantity.[8]

Implications:

Pattern and patterning activities are pervasive in the world around us and are central ideas within mathematics

Children are likely to be introduced to three broad types of pattern in school mathematics: repeating patterns; spatial structural patterns; and growing patterns

2 Research suggests a positive association between early pattern knowledge and later mathematical attainment, particularly in creativity[9] and algebraic thinking.[e.g., 1] However, the nature of such a relationship is complex, involving: spatial thinking (using visual imagery and/or mental manipulation);[10] visualisation (creating mental images of visual or spatial information);[11] working memory (a short-term mental system that manipulates temporarily stored information related to a task);[12] and fluid reasoning (logical thinking and problem solving in new contexts).[12] The positive association with algebraic thinking may be partly because carefully chosen pattern activities can encourage students to attend to mathematical structure and related relationships, supporting development of symbol and structure sense, generalisation and the use of variables.[1, 13]

Implications:

Early pattern knowledge may have a positive effect on later mathematical attainment, particularly algebraic thinking, through the development of symbol and structure sense, generalisation and the use of variables

The factors relating early pattern knowledge to later mathematical attainment include spatial thinking, visualisation, working memory and fluid reasoning

3 Common patterning tasks in school mathematics often focus on creating, *duplicating* (copying/recreating a visible pattern), and *extending* (continuing a visible pattern) patterns.[1, 12, 14] Duplicating and extending tasks may be carried out using the same materials/objects as the original pattern, which is generally considered easier,[1] or by using different objects to represent the original pattern – a process called *abstraction*. Of the two, pattern abstraction is the more challenging, but is highlighted as an important focus for activities with young and even pre-school children,[1] as these tasks require attention to the overall structures of the pattern and a unit of repeat (motif). Pattern abstraction has been described as an early example of algebraic thinking, as the act of "using the same name for patterns with different physical elements [...] is in essence naming a variable".[13(p174)] It is suggested that describing what stays the same and what changes within and between patterns (see infographic) supports pattern abstraction.[15] This is further enabled by introducing common verbal labels (letters in particular, e.g., ABABAB ...) that can be used to refer to elements across different patterns.[1]

Implications:

There are three main ways to interact with pattern – creating patterns, duplicating patterns and extending patterns – and students should have experiences with all three

Representing a pattern using different materials/objects from the original is a challenging yet important activity for children of all ages, as it supports algebraic and pre-algebraic thinking

Introducing common verbal labels, such as letter labels (ABABAB ...) that can be used across different patterns, supports abstraction

4 Informal exploration with building up and breaking down physical and visual patterns can support students in uncovering a generalised description or rule.[16] This is especially important when thinking about growing patterns, rather than solely focusing on numerical data and algebraic representation.[16] When children are encouraged to notice what stays the same and what is changing as part of exploration and sense-making with pattern, they are likely to develop an understanding of the underlying structures.[15] It may be helpful to use visual growing patterns that can be "chunked into recognisable parts"[17(p308)] and reconstructed using physical materials.[16] Using number cards, which can be physically placed alongside a sequence of figures, can support students to keep track of ordinal positions when comparing elements.[18] In this way, children can also think about how changes in the elements of a pattern and elements of the counting sequence are coordinated.[18] Such activities may encourage an important shift towards a view of pattern that is related to mathematical functions.

Implications:

Playing with visual patterns by decomposing them into recognisable parts supports students to uncover generalised rules

Encouraging students to look for and describe similarities and differences between pairs of elements in a pattern can draw attention to a general structure

When playing with pattern and order, using position number cards can help students to associate each element of a pattern with its position

> "Patterns are the heart and soul of mathematics"
>
> Zazkis & Liljedahl, 2002[6(p379)]

> "Even for professional mathematicians the beginning of a research project is playful exploration: contemplating patterns, playing with ideas, exploring what's true, and enjoying the surprises along the way"
>
> Su, 2020[19(p53)]

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