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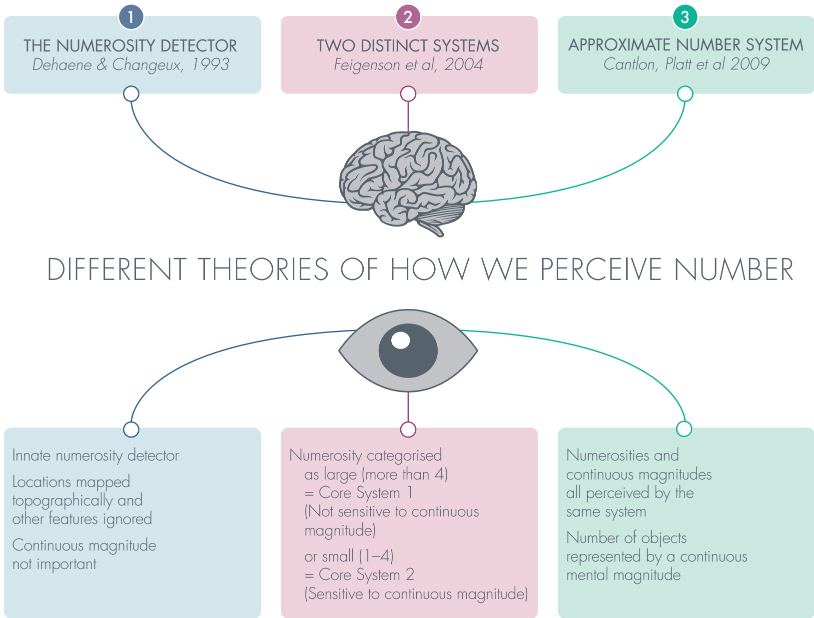
TALKING POINT: WHAT IS 'NUMBER SENSE' AND HOW DOES IT AFFECT MATHEMATICS LEARNING?

'We also know from research that a key focus for early mathematics is developing number sense, especially understanding number symbols, e.g. 'the fiveness' of 5'

(Advisory Committee for Mathematics Education)



1

Number sense is a term that is defined differently by different experts – there is particular discrepancy between the two fields of cognitive science and mathematics education (Gersten, Jordan & Flojo, 2005). Kalchman, Moss & Case (2001) suggest there is some consensus on a definition that includes: fluency in estimation, ability to recognise reasonable results, flexibility when mentally computing, ability to move among representations and choose the most appropriate. Okamoto (2000, in Kalchman, Moss & Case, 2001) defines it as the two key skills of counting and quantity discrimination (comparing) – and crucially, the link between these two; these are the precursors to all the other elements. In this document, we use a combination of these two ideas to define number sense: in other words 'the ability to perceive, manipulate and understand numerosities' (Cantlon, Platt et al. 2009; Feigenson et al. 2004 in Leibovich, in press), which appears to be the most popular and well-documented definition.

The findings from experiments largely agree, with the disagreement often being about the models underpinning observed behaviour (see infographic; Sarnecka, 2015). While some researchers consider the origin of number sense a 'genetic endowment', others think it is an 'acquired skill set' (Berch, 2009), but almost all agree it can be improved through teaching in some way.

IMPLICATIONS: Number sense is not a term that all researchers define in exactly the same way, but includes basic counting and comparing skills and the flexible ability to compute and represent number. It can be improved through teaching.

2

Number sense is a foundation for all higher-level mathematics (Feikes & Schwingendorf, 2008) and a conceptual structure that relies on links between mathematical relationships, principles and procedures (Case, Harris & Graham, 1992). The maturity and efficiency of early counting strategies are predictors of students' ability to profit from mathematics teaching (Geary, Hamson & Hoard, 2000) and there are some key early concepts which correlate with later maths achievement: quantity discrimination, identifying the missing number in a sequence, and number identification, as well as working memory (Gersten, Jordan & Flojo, 2005).

IMPLICATIONS: Number sense is a foundation for higher-level mathematics and correlates with later achievement in maths.

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3

Early number learning seems to follow a very well-defined pattern, with clear stages. Infants can show surprise when the number of objects in a set changes unexpectedly, which demonstrates very early ability to count (eg Feigenson & Carey, 2003). Young children learn to recite numbers first without understanding their meaning, then they begin to fill this 'placeholder structure with elaborated concepts' – known as 'conceptual boot-strapping' (Carey, 2009). Young children can directly recognise (subitise) up to three objects, which usually increases to four with adulthood; they must understand the 'cardinal principle' – that the last word of a count is the total number of the set – before deeper conceptual number sense occurs (Sarnecka, 2015). Children from lower-income families tend to go through these developmental stages significantly later than children from higher-income families (ibid).

IMPLICATIONS: Research on number sense shows clear universal stages of development in early learning about number, which can help teachers to plan and assess.

4

If number sense is viewed as a skill set as opposed to something intrinsic, 'it should be teachable' (Gersten, Jordan & Flojo, 2005), although some researchers suggest good number sense 'cannot be compartmentalised into special textbook chapters or instructional units (Verschaffel & DeCorte, 1996) and its development does not result from a selected subset of specifically-designed activities – it may be better to see it as a 'by-product of other learning rather than a goal of direct instruction' (Greeno 1991). Good number sense appears strongly linked to high socio-economic status and therefore may also be linked to informal instruction at home; helping some students with specific aspects of number sense like quantity discrimination may help them quickly catch up with peers (Gersten, Jordan & Flojo, 2005). Effective use of board games with young children before they go to school can enhance early number sense (eg Siegler & Booth, 2004) and in the first years of school helping children make links between verbal and symbolic, digital and analogic representations of number can help develop number sense (Kalchman, Moss & Case,2001).

IMPLICATIONS: Number sense is something that can be improved, although not necessarily by direct teaching. Moving between representations and playing games can help children's number sense development.

IN SUMMARY

- Number sense is defined by counting, comparing and flexible thinking about number, although some researchers don't agree on the exact definition
- Number sense is a foundation for maths learning
- Tests for aspects of number sense correlate strongly with later mathematical achievement
- Preschoolers generally learn about number in clearly defined stages
- Good number sense appears linked to high socio-economic status and these children also tend to go through developmental stages earlier
- Number sense can be improved by helping children make links and move between representations; board games can help

'People with number sense are those who can use numbers flexibly'
– Jo Boaler, Stanford University

'Rapidity doesn't have a precise relation to intelligence. What is important is to deeply understand things and their relations to each other'
– Laurent-Moise Schwartz, mathematician

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