CAMBRIDGE Mathematics

Methodology

Research-informed design

Others in this series

- Building the research base
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Methodology: Research-informed design

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Introduction

In this paper we discuss the rationale for our design approach and the inputs, outputs, methods and processes it comprises. We position our design methods and influences within educational design methodology more broadly and provide descriptions and examples for each component of our approach. Throughout the paper we connect our design methods to other aspects of the Cambridge Mathematics Framework project, including our approaches to research, formative evaluation and ontology development. We also connect our current work back to the overarching goals and fundamental principles laid out in the Manifesto for Cambridge Mathematics and its most recent update.

Background: Design in education

In education, design is an approach to creating tools, objects and processes which make some useful contribution to the field. The process may also be conducted as a form of research, making a contribution to theories of teaching and learning. Methodology for design and design research in education has been gradually developed into a set of more systematic and well-theorised approaches over time (Cobb, Confrey, diSessa, Lehrer, & Schauble, 2003), with a strong focus on supporting positive impacts on teaching and learning (Barab, 2014; Burkhardt, 2009).

Design projects share the following general components, implicitly or explicitly:

- *Design goals* describe what aspects of the problem the design is intended to address.
- *Design principles* describe approaches to meeting particular design goals which will then be implemented in the design.
- *Inputs* might be influences, reference examples, prior knowledge, existing information, or initial discussions – or they might be outputs from formative evaluation or a previous design process.
- *Outputs* might be components of the overall design, components of formative evaluation (like research protocols), design principles, or agreed design practices.
- *Processes* may be developed for the purpose of "investigation/analysis; design/prototyping; evaluation/retrospection" (McKenney & Reeves, 2012, p. 76).

Our process of developing *design goals* and *principles* for our design context is discussed below. Our *inputs, outputs* and *processes* are shown in Figure 1 and discussed throughout this paper.

Because educational designers aim to bring about positive change in some aspect of educational practice, the design process is typically

- *collaborative*, to involve those whose roles in education mean their professional experience is embedded in the context of the problem (and the solution),
- *responsive*, able to course-correct on the basis of formative evaluation as design progresses, particularly on the basis of pilot testing as the design reaches the point where some part of it can be implemented with its intended users,
- *iterative*, because once a design is evaluated in some way, the data may suggest it should be adapted or refined, and
- in the case of design research, *theoretically grounded*, so that the design can be informed by theories built on prior empirical work in relevant areas and can make a useful contribution to theory (McKenney & Reeves, 2012).

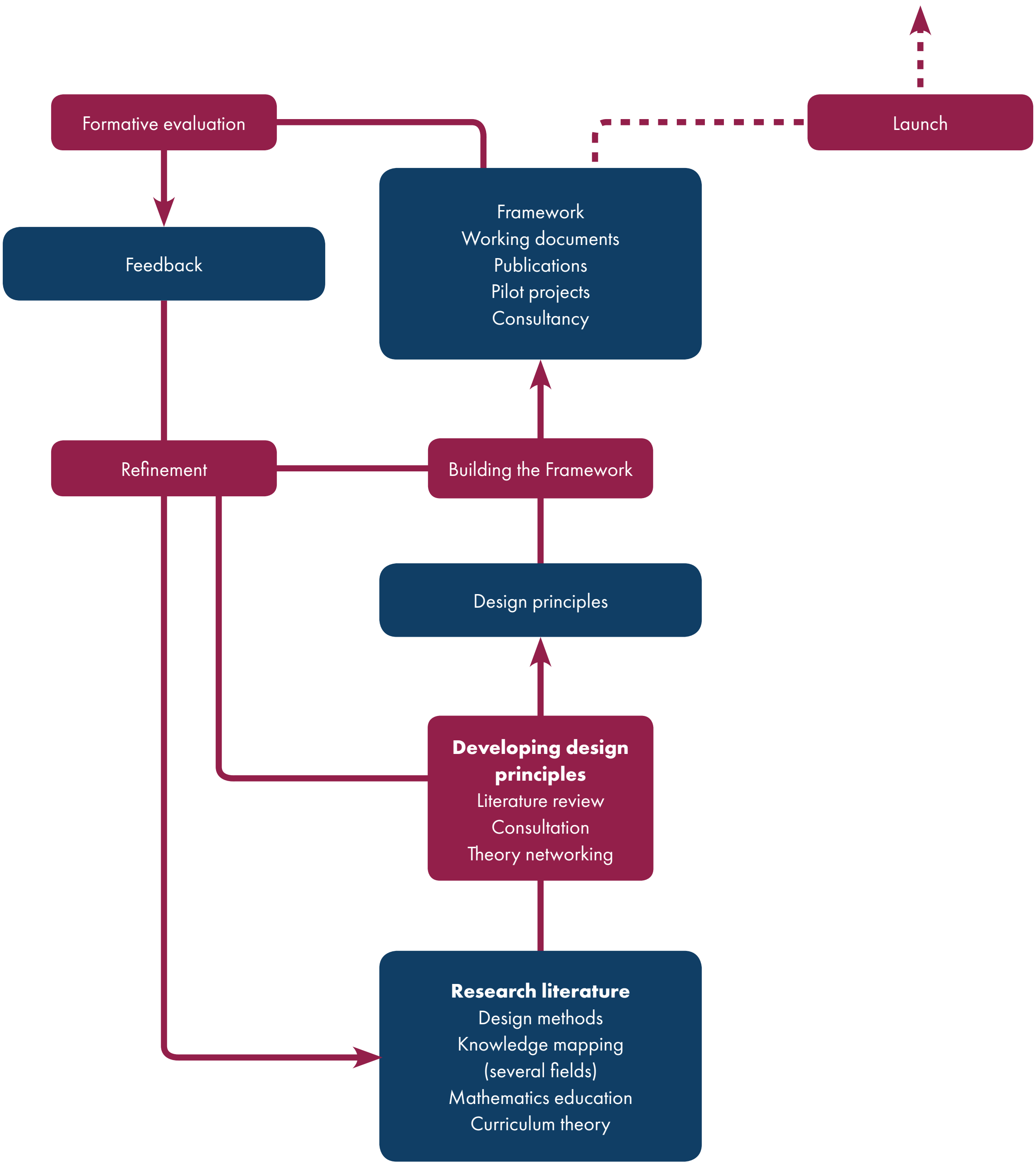
Through *design cycles*, components of a design may be refined to be more effectively aligned with their purpose. Feedback from implementation or contact with stakeholder communities might be the driving force for refinement and can take place as soon as appropriate feedback is available, which might occur on multiple different scales of time within a single project. Our major design cycles, for example, are shown in Figure 1, which indicates that all of them so far have occurred before the design is complete.

Each major area of decision-making in design – determining the design problem, the design process and the design solution – occurs over time and may be informed by research (McKenney & Reeves, 2012). The *design trajectory* is the path of development of a design which is determined by these decisions. The *design narrative* is a record of that trajectory and the basis for decision-making along the way, which may itself contribute to the development of design principles and/or future research. Figure 1 shows elements of the cycles which have formed our design trajectory so far. Our processes for research-informed decision-making are discussed below and in other related documents on our website[1].

[1] See Methodology: Building the research base (Jameson, 2019), and Methodology: Formative evaluation (Jameson, 2019)

Design methods: Cambridge Mathematics Framework

Figure 1: Main design cycles in the Cambridge Mathematics Framework design process (inputs and outputs in blue, categories of processes in red)



This paper focuses on our design methods, the processes we follow within those methods and major inputs and outputs of these processes. We have developed our design methods in relation to our design goals and the design problem context, which are described elsewhere[2].

The research landscape

Design projects in education can have different orientations for their research activities. It is useful for us to make a distinction between *research-informed design* and *design research* in order to demonstrate where our design project sits and to explain our approach.

Research-informed design incorporates influences from existing research. When design is also intended to make a contribution to theoretical development in the field, it is *design research*, which may be called by different names depending on the influences and approaches of the designers (Barab, 2014; McKenney & Reeves, 2012). Both share many characteristics: they may be informed by existing theory and data in the same ways, involve many of the same attitudes, values, methods, and outcomes, and may gather data to inform iterative refinement of a design. They may contribute design principles which bridge theory and design in a way that could be useful for other design projects. However, specific design choices in *research-informed design* are shaped only by the contribution to some form of practice in education, while in *design research* these choices must also support theory-building (McKenney & Reeves, 2012).

The Cambridge Mathematics Framework design approach

Our approach is primarily *research-informed design*, because our approach is structured by the goal of producing a tool that can help to address problems in mathematics curriculum design and enactment, rather than the goal of developing educational theory. We focus on the productive application of existing theory and empirical data to our design. In this way our use of theory has been predominantly consistent with a *design as intention orientation* (Collins, Joseph, & Bielaczyc, 2004); that is, making the initial design both internally coherent and consistent with well-defined theories and the professional experience of the designers informing it (Ruthven, Laborde, Leach, & Tiberghien, 2009). Even so, in order for our work to be as effective, transparent and useful as possible, we have incorporated some traits into

[2] See An update on the Cambridge Mathematics Framework (Cambridge Mathematics, 2018)

our approach which McKenney & Reeves (2012) classify as being more often associated with *design research*. We

- analyse our design and formative feedback from evaluation according to frameworks derived from literature review,
- formally document our aims, goals, research influences, methods and results for a wide audience, and
- seek to learn from other design efforts with similar goals and/or contexts.

This approach leads us to be more prepared to elicit and receive formative feedback from a range of relevant audiences, and we are better able to weigh available options and the consequences of our choices at every stage of design.

Developing the problem context, design goals and design principles

Problem context

The nature of the problem to be solved shapes determination of the design goals and the design principles which help to realise the solution. Our design project began with the research which led to the writing of A Manifesto for Cambridge Mathematics. That document represents the initial exploration of the context for the design of the Cambridge Mathematics Framework. It lays out background knowledge of the problem context from the professional experiences of the Director and Cambridge University partners of Cambridge Mathematics, senior curriculum designers and researchers whose work has had national and international impact, and blends in some additional research which has served as a starting point for design. We continue to refine and augment our perspectives on the problem as additional research and feedback contribute detail and examples for decision-making. Some of these developments have been reported previously[3]. Others will continue to be included in upcoming reports and papers as appropriate to the topic.

Design goals

Design goals are developed from an understanding of the problem context and the intended scope of the design project. These define the outcomes the design is being engineered to support in terms of their contribution to solving the overall problem. Initial design goals are set out at the beginning of a project, but may be added to or adapted as experience with the particular design and/or context increases.

[3] See current examples in the An update on the Cambridge Mathematics Framework (Jameson, McClure & Gould, 2018), Shared perspectives on research in curriculum reform… and reports in our Framework Documentation series

Our initial design goals were laid out before design work began and in our case, because the goals themselves came from discussions involving a great deal of practical experience with the problem, the goals themselves have not changed appreciably. What has changed as work progresses is that our intermediate design solutions for matching scope and resources have evolved as we have developed and implemented our design processes, and as a result our sense of what it looks like to realise our design goals in the short, intermediate and long term has improved.

Design principles and specific translations of design principles

Once design goals are defined, discussion moves to how the design will meet those goals and what features it needs to have in order to do so (Sandoval, 2004). *Design principles* are interpretations of theory and background knowledge of the design context which lay out general perspectives on how to approach the design. They guide design choices about the features a design should include and the functions those features should support (McKenney & Reeves, 2012). They can also help to make a particular design process more transparent, providing justification for specific design decisions, features and affordances.

Explicit design principles, and decisions about how they will be enacted, arise from a process of reflection – on theory, experience, design goals and data from implementation if available (McKenney & Reeves, 2012). Initially, this may be designers' reflection on their own initial assumptions, knowledge of relevant theories and relevant professional experience. Starting assumptions can develop or change as they are explored and challenged throughout the design project, and so design principles may be changed and improved, prioritised or deprioritised as a project develops. They may also emerge explicitly as principles at some point after they begin to be used in the design, as the act of designing can help designers to recognise some of the ideas and influences which have been implicitly guiding their work. Our design principles have certainly emerged in stages, as described below.

Not all design projects make distinctions among types of design principles. Designers may apply design principles implicitly without specifying them at all, let alone how they were formed or how they link theory to design. Differences in the scope and specificity of design principles affect how straightforward it is to apply them (McKenney & Reeves, 2012; Sandoval, 2004) as well as how much interpretation and how many interrelated theoretical influences will support well-informed decisions about the design. Theories are typically not developed for precisely one context and designers must find ways to bridge the gap successfully.

The term *specific design principle* describes those principles which apply more narrowly to specific design features and tend to be informed by relatively localised theories (Linn, Davis & Bell, 2004, as cited in McKenney & Reeves, 2012). Similarly, Sandoval (2004) uses the term *conjectures* to mean interpretations of theory which are specific enough to be directly embodied in design. The word conjecture rather than principle places an emphasis on the role of the conjecture in empirical refinement of theory for the sake of design research. In our case, for research-informed design, the ability to trace theoretical influences to specific design features helps us to improve the design and design principles even from a purely functional perspective. It becomes easier for us to pinpoint which assumptions we should revisit when we realise we need to change something and easier to do additional research and evaluation to support refinement (see Figure 2).

In our process of Framework design the kind of design principles which have evolved most significantly over time are at the more design-specific level, because it is at this level that our experience from research and pilot implementation of the design is growing the fastest. Even so, some additional general design principles have emerged as priorities. Our original and emergent design principles are listed below.

As our design trajectory unfolds, there are times when ideas for general design principles collectively come to the foreground and we articulate them through team discussion. In some cases this is due to recognising an unmet need to communicate certain aspects of the project through conversations with our audiences, which in turn provokes us to better characterise these aspects for ourselves. At other times the process is a more deliberate revisiting of assumptions when we recognise our understanding has progressed.

This practice of formally identifying specific design principles or conjectures after the fact is well recognised (Sandoval, 2004), as it is then easier to see what is most important to formalise. When the design is implemented, users of the design put it into action and designers can answer these questions:

1. What is being done with different elements of the design, and is it all functioning as intended?
2. What happens as a result; does the design help users achieve their larger goals?

Designers can then work backwards, observing what actions users are taking to achieve their goals, identifying what elements of the design enabled these actions, and finally referring to previous evidence to build a description of the most fundamental specific design principles and their justification.

Developing the rationale for specific design choices

Sandoval (2004) calls the specific translations of design principles involved in (1), above, *design conjectures*, and in (2) *theoretical conjectures*. He introduces the process of *conjecture mapping*, in which selected design features are mapped back to conjectures and underlying theoretical influences, and mapped forward to user actions and outcomes. This helps designers to reflect on the logic of the design as a whole and the output – the conjecture map – is an explicit model of the logic of the design. This serves as a *feature-specific type of logic model* which can then provide guidance for refining the design if necessary. While it is not our goal to develop conjectures which can make a direct contribution to theory, we have found this mapping activity to be useful when developing and analysing our pilot cases, and the resulting logic model has helped us to communicate more clearly in our ongoing internal and external design discussions.

For our pilot case studies, we have adapted the idea of conjecture mapping to create feature-specific logic models for analysing specific aspects of Framework design. While a typical logic model provides a more general overview of impact through design (Greene, 2013; Newton, Poon, Nunes, & Stone, 2013), these feature-specific logic models help us to

- identify, though observed user actions, our most significant design features and functions and the mix of influences behind them,
- develop and refine specific translations of design principles (e.g. adapt tools to make important actions easier for users),
- articulate our rationale for specific aspects of the design,
- plan appropriate formative evaluation, and
- further develop our model for impact; that is, the Framework's contribution to improving the design and/or implementation of mathematics curricula and supporting materials.

Our feature-specific logic models describe aspects of the design in terms of the categories shown in Figure 2. Figure 3 provides an example of a portion of the model for textbook chapter authoring, one of our early pilot cases. Development and refinement do not always occur in the order shown. Sometimes design features are suggested through experience or intuition, and we then go on to develop an understanding of their theoretical context. In other cases, we see users developing techniques that help them to make better use of the Framework and we develop design features to support them. Through our formative evaluation process we gather evidence, prior to full implementation, that the Framework shows promise for supporting positive outcomes.

Figure 2: Components of our feature-specific logic models (adapted for our process from Sandoval, 2014)

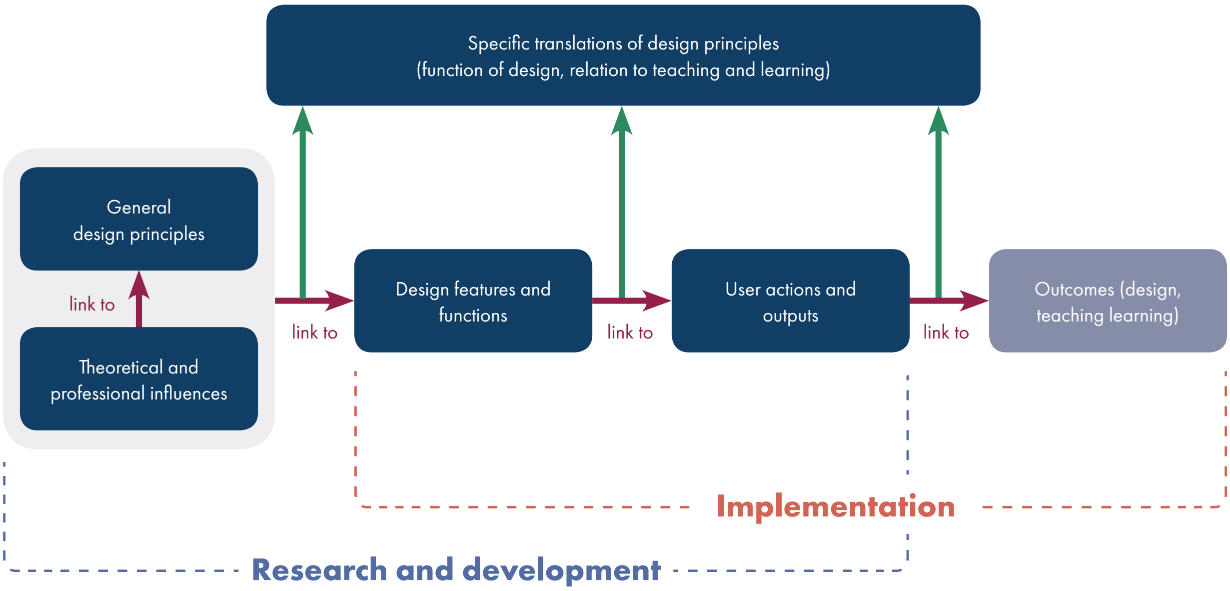
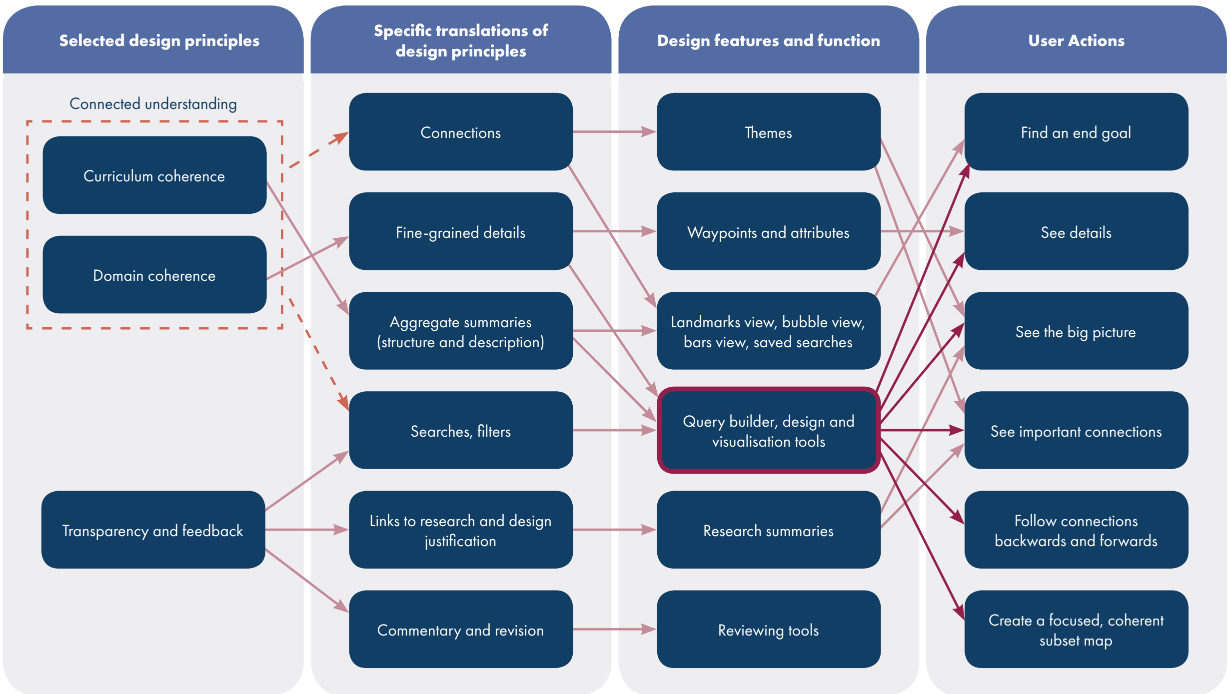


Figure 3: Example of a feature-specific logic model derived from analysis of a case in which use of the Framework was piloted for writing a textbook chapter. The user output (not shown) was the textbook chapter.



Managing design influences: Networking theories and practices

What problem in research-informed design does networking theories solve? What is our guiding question?

The process of building Framework content involves the synthesis of a large number of influences from the literature, collaboration and professional experience. These may be more general or more specific to the topic at hand, may have been developed and used for a variety of purposes, and may not always be clearly named or defined in the sources that make up our literature reviews. Our research base in particular is heterogeneous because we need to draw on research not only about mathematics teaching and learning in specific topic areas but also from other fields within and beyond education which can inform our design approach. These include

- design processes,
- the extended design which includes and supports the Framework, and
- our formative evaluation approach, which feeds back into the design process.

Explicitly mapping every influence to the thousands of specific decisions about content and structure would be prohibitively complex and of questionable utility. Instead, we write *Research Summaries*[4] to explain how theory and evidence from research have influenced our work. The process of synthesising research usually involves *networking theories and practices* in some way. In doing this one of our most important guiding questions is: How can our design express theories and evidence from research meaningfully and coherently as a set of features, tools and processes?

Why is it valid to consider networking theories and practices?

When the goal is design, working with multiple theories and practices, which Sfard calls "theoretical pluralism" (2003, p. 355), can be beneficial. It is often necessary when design outcomes depend on anticipation of complex contexts in the real world (Kieran, Doorman, & Ohtani, 2015; Prediger, Bikner-Ahsbahs, & Arzarello, 2008). Multiple theoretical perspectives should be brought to bear on a design in order for it to be successful, as most theories are only constructed to look at a particular phenomena a design may involve, and many theoretical perspectives are either complementary or talking at cross purposes rather than truly contradicting each other (Sfard, 2003). In contrast, when the main goal is research, then specific sets of theories, research questions, and research practices are developed to

[4] See Methodology: Building the research base

study specific phenomena as clearly as possible, often in a deliberately simplified theoretical context (Artigue & Mariotti, 2014).

A successful design is the result of decisions enabling it to accomplish some things and not others, but even a focused design relies on appropriately-informed assumptions. The Framework is a network of mathematical ideas, which in turn can be tied to teacher education and training, tasks and assessments. It is designed for specific uses related to mathematics curriculum design and implementation. Table 1 presents particular areas of the literature providing theories, data and practices we draw on to inform our design.

Table 1: Areas of the literature informing our extended design

Aspects of design	Where networking theory and practice takes place	Areas of the literature
Mathematics teaching and learning, topic-level	Research summaries; Framework content and structure; Espressos, blogs; Meetings and discussions	Mathematics education
The overall design and design process	White papers, reports; Conference papers; Publications; Meetings and discussions	Mathematics education; Learning Sciences; Computer Supported Collaborative Learning (CSCL); Computer Supported Collaborative Working (CSCW); Information Science (IS), Human Computer Interaction (HCI), and User Experience (UX)
Formative evaluation	Formative evaluation methods and instruments (survey and interview); Meetings and discussions	Mathematics education; Qualitative methods in the social sciences

These areas of literature involve different, though sometimes overlapping, *systems of research practice*, defined by Artigue & Mariotti (2014) as consisting of underlying theories, research questions and topics, research designs for addressing these and methodologies providing justification for the research designs. Prediger, Bikner-Ahsbahs, & Arzarello (2008) similarly identify these components as core ideas and assumptions, empirical elements and the area in which the research can be applied. When networking theories and practices these systems may be linked to each other by one or more of these components

(Prediger et al., 2008). Within the themes shown in Table 2, studies often hold many philosophical, theoretical and methodological influences in common.

Table 2: Example themes in the literature from which we have networked theories to inform design and formative evaluation strategies

Themes in the literature	Design goals, principles, features	Evaluation
Boundary objects, boundary negotiating artifacts (BNA); Knowledge sharing, collaborative knowledge building; Expert knowledge in communities of practice	Shared artifact (Framework maps), explicitly stated and structured content: • Flexible, shareable arrangement of content in explicitly linked maps • Alternative display of content as simplified table	Pilot case studies; Delphi study; Interpreting feedback
Audiences, signals and indicators	Create pieces which can be meaningfully experienced by various audiences	Provide settings for audiences to engage with the design, collect feedback and use it to inform the Framework
Typology of experts, including knowledge brokers	Contributes to user profiles interface requirements	Interpreting pilot cases
Expert knowledge in communities of practice	Same as boundary objects	Delphi Interpreting feedback
Knowledge perspectives	Designers can see some of what is useful for teachers	Pilot case studies – is it happening? Delphi study – is there any discussion of it?
User group professional knowledge and ways of knowing (teacher, designer etc.) • Content knowledge • Pedagogical content knowledge • Design knowledge • Epistemic beliefs about mathematics and mathematics learning	Teachers and designers can get a wider-horizon design perspective • *Waypoint* descriptions and *Student Actions* • Professional development layer • Tools for use	External review; Delphi study; Pilot case studies

In our case, different aspects of the Framework require us to coordinate informed assumptions about

- the nature of mathematics learning, including what perspectives have been identified as essential and in need of more support, i.e. what our model might highlight,
- the nature of professional knowledge of mathematics learning and how it might be coordinated across multiple professions,
- the nature of design and how we should carry out our work in order to achieve our goals for the quality and usability of the framework.

What are we networking? Theories in education

The nature of theories is what allows them to be usefully networked and applied together. Based on a review of perspectives on theory in the mathematics education literature, Artigue & Mariotti (2014) present an integrated perspective on theories as systems which shape research practice, described above. These systems of theories can then be combined in systems-of-systems, whether to inform research or design projects, or to develop new theories. We do this in our design, as Table 2 illustrates, because different aspects of the design relate to different types of phenomena in the real world. The details of communicating knowledge of mathematics concepts to specific professional communities are very different from the details involved in creating an interface for generating knowledge maps. If we want to learn from work which has come before us, we need to seek it out in each appropriate area.

The process of combining theories can be complex but muddy. Broadly, education as a field is applied and interdisciplinary with fundamental social implications, and as a result theory can sometimes be difficult to distinguish from philosophy (Siegel, Phillips, & Callan, 2018). In our metadata for the literature reviews in our research base we have certainly noted sources[5] for which this distinction is not made clear. Nevertheless, the theoretical perspectives in these sources contribute to our work.

The scope of a theory is a necessary consideration when considering how theories are related to each other in mathematics education. Kieran et al. (2015) and Ruthven et al. (2009) discuss the role of grand, intermediate, and domain-specific frames. *Grand-frame theories* (e.g. constructivism) shape fundamental perspectives on learning but are not directly applicable to the specifics of design. *Intermediate frames* have a narrower focus whose origins may be "primarily theoretical or…based to a large extent on deep craft knowledge" (Kieran et al., 2015). They tend to have implications that can be more clearly and directly applied to our design decisions given our design-with-intention orientation.

[5] As described in Methodology: Building the research base

Conversely, they can also serve to link theoretical interpretations at the level of individual studies to grand-frame theories (McKenney & Reeves, 2012). *Domain-specific frames* are even more narrowly centred on particular concepts and processes (Kieran et al., 2015). Finally, McKenney & Reeves (2012) use the term '*local theory*' to describe the theorising designers do to make sense of or predict what happens in the implementation of a particular design in a particular context. On this scale, interpretation and use of data from implementation is aided by linking local theories to wider but still domain-specific frames, intermediate and grand frames (McKenney & Reeves, 2012).

In the design of our Framework grand frames help to shape our overall orientation, which influences which intermediate frames we work within most and consequently how domain-specific theories influence Framework content and structure. For example, constructivism and activity theory (as grand frame learning theories) influenced our ideas of representing waypoints in a dynamically generated map. They also influenced the Swan task design framework (Swan, 2014) (as an intermediate frame theory), which has influenced the structure of Framework features like waypoints, while the content of those waypoints is influenced by a host of domain-specific theories. We also use intermediate or grand frames to help us make sense of whether two domain-specific theories conflict or are commensurate from different perspectives, which helps us to synthesise the story of a set of waypoints from our literature reviews for each Research Summary[6].

How can it be done, and how are we doing it?

Prediger et al. (2008) identified a range of strategies of the practice of networking theories in mathematics education:

1. Not connecting to other theories
2. Understanding and being understandable to other theoretical perspectives (translating language/ expression of theory or problem context)
3. Contrasting and/or comparing to other theories
4. Combining or coordinating with other theories
5. Synthesising other theories for localised integration
6. Weaving everything into a global theory

[6] Framework features are described in more detail in Methodology: Building the research base

The most extreme strategies, connecting nothing (1) or connecting everything (6), are the least practical or useful for a typical design or applied research project. We use comparing and contrasting (3) occasionally in Research Summaries but not as much in other areas of design; these are most commonly used in the research as strategies for discussing the rationale for choices of theoretical influences for research design (Prediger et al., 2008). We predominantly use combining and coordinating (4) strategies in our work; these help a researcher or designer to better understand multiple relevant aspects of a complex phenomenon or construct (Prediger et al., 2008). For us, this also entails doing some translation (2) between theories and practices in different research contexts (see Table 1). Synthesising theories (5) is a more formal theory-building process which is outside the scope of our present work.

Our review of research and the translating, coordinating and combining of theories and practices takes place in different parts of our overall design process (see Table 1). We bring ideas from individual reading to team meetings, try ways of incorporating them in design and discuss the results. We write up our literature reviews in Framework-embedded Research Summaries for topic-specific areas, in *Espressos* (short research digests for teachers) and blogs for professional development, in reports, conference papers, and publications for our overall design and methodology, and in our survey and interview methods for formative evaluation. The act of reviewing and writing itself serves to deepen and formalise the unfolding theoretical landscape of the project. Discussions and feedback around the writing extends this process once others engage with it in its published forms. These processes are cyclical in that, as we continue our literature reviews and receive feedback on current work, we continue to integrate additional theories and practices when appropriate. Finally, we use techniques like a variation on *conjecture mapping* (described below) to record explicit links between theories, design features and emerging uses as parts of the design are implemented in pilot cases.

Is our networking of theories and practices valid?

As described in Methodology: Formative evaluation, our theoretical influences and justifications, along with their links to our design, are reviewed as part of a set of formative evaluation processes. These include external review of the Research Summaries and related Framework content and expert panel review of Framework structure in a Delphi study. We design our survey instruments to include evaluation of our uses of theory, data and research practices from the literature and make changes based on feedback.

At what point in the process should this take place?

More formal networking of theories and practices is often done after work has already begun (Artigue & Mariotti, 2014). As we network theories and practices in our work, we often begin from a starting point in our literature review and layer in more influences and justifications from research as we proceed.

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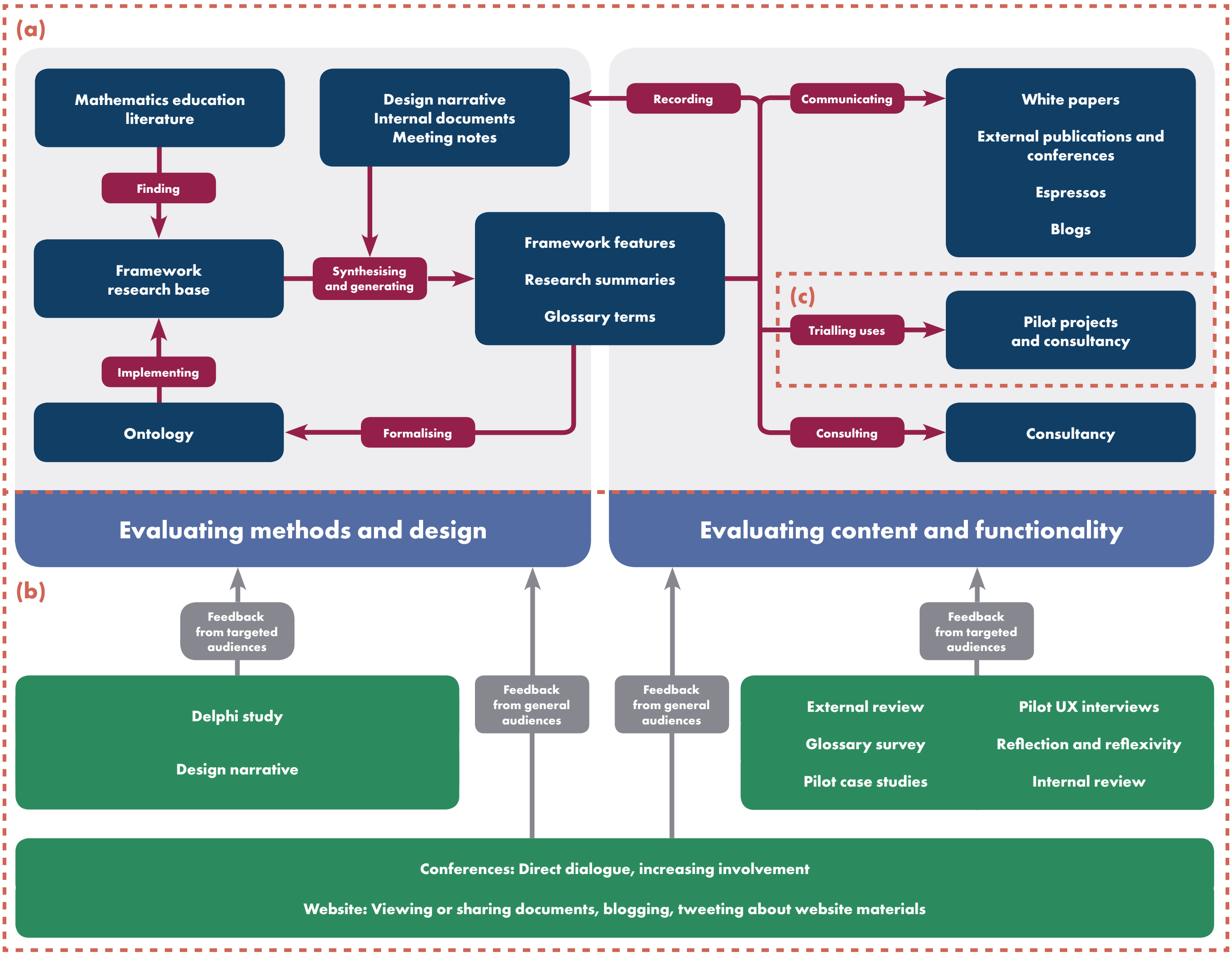
In writing the core Framework content, we have come upon different ways of looking at connections between mathematical ideas along the way which have inspired changes to the content itself, how relationships between mathematical ideas and experiences are structured, how they might be connected to other curriculum elements (e.g. tasks, teacher professional development, or assessment) and how they can be made accessible as part of the connected whole of the Framework.

Building the Framework

The extended design: a summary of Framework components and design processes

Figure 4: Components of the Framework design process showing (a) the extended design, (b) formative evaluation and (c) pilot implementations.



As the structure and background of the design project took shape, we were able to start designing and building the Framework itself. In the typical way of design projects, the act of starting to build the Framework helped to clarify our understanding of the structure and background of the project as a whole, and the set of components of the *extended design* (the Framework, internal and external documents, and practices for pilot implementation and feedback) grew to include everything shown in Figure 4 (a). Of these, literature review, the research base and preliminary Framework features came first, as we describe in Methodology: Building the research base, along with initial ideas for features of the website. The other components and processes emerged from needs which became apparent as our work progressed.

There have been some notable cycles which have led to iterative development and refinement of our work. For example, the emergence of components like the ontology, which formalises the synthesis of content of the Framework, was linked to the move towards putting content into a database which would enable designers to keep track of connections as well as the specifics of content. This spawned a host of new design features and specific design principles, all of which drove revision of some of the prior content and the ontology itself and led to additional formative evaluation strategies. Feedback from these evaluation strategies in turn has led to further revisions.

Integrating design principles

As these design cycles continue over time our understanding of the bigger picture develops further, along with our priorities for the design. Table 3 shows that the design principles we identified at the start of the project have been added to over time, as the act of designing helped us to consciously designate priorities emerging as core principles out of the wider space of possibilities. Work done on the basis of three of our initial design principles[7] led to adapting one and identifying three more as we explored feasible directions for the design[8]. These new design principles were more specific to the direction in which the project was going. Most recently, now that we have been able to observe the Framework in use, the flexibility afforded by our search and visualisation tools in combination with the ontology has emerged as a core design principle as well, helping us to support it further in our decision-making. In all cases, taking the step of articulating a priority as a design principle has guided day-to-day internal design discussions and helped us to develop questions for formative evaluation.

[7] Laid out in A Manifesto for Cambridge Mathematics (McClure, 2015)
[8] Described in An update on the Cambridge Mathematics Framework

Table 3: Design principles integrated at different stages of design

Initial	Access to mathematics for all students Collaboration and consultation Support for a coherent programme Evidence-based
Added	Research-informed (adapted from evidence-based) Transparency Connectivity Early experiences
Recently emerged	Flexibility

Consultation and discussion: Participatory design and collaborative knowledge building

Participatory design is an approach which involves members of stakeholder and/or beneficiary communities as participants in the design process. It is collaborative and communicative, helping to refine both the design principles and the details of how they are enacted. (DiSalvo & DiSalvo, 2014). While it can be true that "too many cooks spoil the broth," the design process can be structured to maintain coherence amidst competing perspectives. In our design process we have developed different structures for different levels of group discussion:

- Individual members of the core design team interact with discussions in the mathematics education research literature.
- The core team meets to discuss both specific and general design issues (see Table 5) and to develop outputs for facilitating discussion with targeted and general audiences. Because of their varied backgrounds, representative of potential Framework users, team members pay attention to different aspects of the design and design methods.
- Targeted audiences are recruited for more extensive involvement in structured discussions. They invest more time and bring additional perspectives on design and research.
- General audiences encounter our work through direct or indirect engagement via specific, public outputs and this helps us to anticipate necessary Framework design decisions for different audiences.

Our methods for bringing feedback from targeted and general audiences into our design discussions are described more fully in Methodology: Formative evaluation.

The Framework is designed for the dynamic creation of knowledge maps which serve as representations of mathematical ideas in curriculum design. These can be used individually or as shared knowledge representations within groups (e.g. instructional design teams) or between groups (e.g. curriculum design committees). We have developed a technology for designing, building and delivering these shared representations.

We take Stahl's (2006) framework for collaborative knowledge building as a useful model for how we develop understanding of and consensus for Framework design. This framework describes an individual cycle of learning, "building personal knowing" connected to a group interaction cycle and "building collaborative knowing" (Stahl, 2006, p. 327). First, an individual participating in a discussion articulates an idea from their background understanding to the group. The statement they make to the group is discussed, involving arguments, rationale, new statements, negotiation and clarification of differences from multiple perspectives of participants in the group. The discussion may result in the emergence of some shared understanding among the group, which becomes part of knowledge held within that group – some of which contributes to personal knowledge of group members as well. In addition to spoken discourse, an individual or group may develop artifacts – text, diagrams or other representations of knowledge – which can persist beyond the discussion and be shared within or outside the group. Table 4 shows how elements of our extended design support knowledge building within our core team meetings, described below and in Table 5, and by proxy in our discussions of formative feedback from our external audiences.

Table 4: Elements of the Framework project aligned with Stahl's phases of collaborative knowledge building (adapted from Table 9.1, Stahl, 2006, p. 207)

"Phase of knowledge building	CM Framework project elements
articulate in words	Team discussions, individual writing of content, Research Summaries, Espressos, white papers
public statements	Framework structure and content
other people's public statements	Research Summaries, Espressos, internal review, external review, glossary survey
discuss alternatives	Compare saved searches; mapping
argumentation & rationale	Research Summaries
clarify meanings	Internal review, external review, glossary survey
shared understanding	Network structure, glossary, Espressos
negotiate perspectives	Saved searches
collaborative knowledge	Multi-role team
formalize and objectify	Ontology
cultural artifacts and representations"	Research Summaries and network content, structure; Espressos; white papers

In our overall design process there are several different contexts in which the knowledge building process plays out:

1. Internal discussions focus on various aspects of Framework design (see Table 5), the outputs of which are internal documents.
2. Consultation might be with individuals with relevant expertise or in the form of group workshops.
3. Several of our formative evaluation methods involve discussion, including semi-structured interviews and a Delphi study which involved mediated discourse in an expert panel.

Table 5: Examples of forms of internal discussion:

Meeting/discussion type	Frequency
Making connections between topic areas	Weekly
Maintaining the research base	Weekly
Ongoing pilot case studies	Weekly to monthly
Big questions (un-agreed design priorities or emerging potential structural elements)	Monthly
Internal review (reliability of ontology implementation)	Weekly
Review functionality (can we do what we need to do; are new features or changes to features needed?)	Monthly
Feedback on draft publications	Monthly

Ontology development

The *ontology* is the guiding structure of the Framework – that is, what ideas can exist within it, how they can be expressed, and how they can be related. An ontology can set the ground rules for a knowledge model like the Framework, which can help later users of that model to access and understand its content. Our process for ontology mirrors the more general design process described above, consisting of cycles in which conceptual objects and relationships are progressively identified, defined and modelled, as ongoing literature review and discussion raises and resolves ambiguities (Fürst, Leclère, & Trichet, 2003). In each cycle we judge whether the ontology helps us to support the design principles we have chosen to shape how the Framework emphasises and expresses mathematical ideas, relative to our design goals (Barlas & Carpenter, 1990). We describe the rationale and method for development of our ontology in detail in a separate paper[9].

[9] Framework Design: Ontology (Jameson et al., 2019)

Once a provisional ontology is adopted, designers can build interfaces, groupings, and hierarchies which make it meaningful and accessible as a shared representation, fuelling further cycles of development through use and discussion (Fürst et al., 2003). We developed a provisional ontology[10] and built a custom platform around it which we used to build these interfaces, groupings and hierarchies further. Building these structures helped us to formalise the ontology, and using them helps us to identify unaddressed needs which contributes to the further development of the ontology.

Building a system for collaborative authoring, querying and visualisation

We developed the *CMF Nexus* platform (Stevens et al., 2019) in response to two problems. One was the problem of sharing, integrating and discussing the most up-to-date versions of Framework content. The other was the problem of being able to store all the content and connections necessary to form a coherent map of important mathematical ideas and being able to filter it to get information back out. CMF Nexus solves these problems for us as designers, and forms the backbone of solutions that will apply to other users.

Sharing, integrating and discussing up-to-date Framework content

Framework content is authored by multiple people whose work is interconnected and is frequently added to and adjusted. This would soon be unmanageable if separate versions of work had to be separately and repeatedly integrated with one another. To solve this problem, we identified three requirements the Framework platform should meet:

1. Each Framework author must be able to access and link to up-to-date versions of content in the whole of the Framework, including domain areas written by other authors.
2. The process of sharing content and connections between team members must be streamlined enough that it does not interfere with writing the Framework.
3. There are particular ways we must be able to view Framework content in design discussions: we must be able to choose which content and connections to view for any given purpose, and we must be able to view detail and the bigger picture as needed.

To meet these three requirements, CMF Nexus has been developed as a system for collaborative authoring, querying and visualisation. In response to requirements 1 and 2, CMF Nexus serves as an interface for the Framework database which can be accessed by many users simultaneously. It

[10] Described below

automatically delivers the most recently saved versions of Framework content by all authors, to all authors, while allowing us to record snapshots of previous states of the Framework. CMF Nexus provides authoring tools which simplify and automate the processes of working with the back-end database manager for data entry, retrieval, visualisation and analysis.

In response to requirement 3, the alignment of CMF Nexus with the Framework ontology and the design of the ontology itself allow us to be able to search for and display meaningful subsets of the Framework, a process we call *surfacing* because it brings particular sets of content to the surface of our attention. All the connections to other content in the network remain intact, but are not displayed unless they are included in the search. This means we can generate particular maps for particular purposes to guide the focus of group discussion. This allows the Framework to serve effectively as a shared knowledge representation in design discussions, and everything produced on CMF Nexus can be shared for discussion through the platform within the team and with wider audiences.

We have found that many of our design principles related to meeting these three requirements align closely with existing design principles developed for similar technologies. Stahl demonstrated that technology for individual or group creation of artifacts could be designed to mediate the negotiation of shared understanding (Stahl, 2006, 2016). He developed a set of technology design principles for supporting the collaborative knowledge-building process. One set of these is particularly well-aligned to our use of CMF Nexus as a professional design tool (Stahl, 2016, p. 243-244):

- "Support synchronous discourse"
- "Support multiuser visualization and manipulation"
- "Support turn taking of construction"
- "Support persistency and review of history"
- "Support open-ended exploration"

Others are aligned to ways in which information is communicated in the Framework generally, including:

- Support integration of multiple relationships between ideas (Stahl, 2006)
- Support for "all information to be uniformly structured with indications of perspective and linking relationships" (Stahl, 2006, p. 102)
- Support for "searching, browsing, filtering, tailoring and linking…reorganizing…indexing and matching" (Stahl, 2006, p. 249)

CMF Nexus has been successful in supporting collaborative knowledge building among Framework designers and in our ongoing pilot case studies, leading to the development of additional tools such as switching between detail and the big picture (zooming in and out), and following connections backwards and forwards. When building the Framework, writers and designers use the platform to create, store and discuss content. Their negotiations result in shared representations of the new shared understanding of the group: e.g. Research Summaries, ontology features, or connections between different authors' work.

Building, storing and retrieving a coherent map of mathematical ideas

The second problem we needed to solve was that of creating and maintaining a coherent map which could emphasise the connections between mathematical ideas. We used paper and spreadsheets for our first explorations, but we quickly reached their limits for our purposes. While the Framework ontology addresses the conceptual dimension of this problem, the practical dimension required us to build tools which allow us to put the ontology into practice. To address this problem, we identified the following requirements for CMF Nexus:

1. We must be able to create, store, retrieve, edit and interpret content and connections efficiently.
2. This must be done in such a way that we can work with meaningful parts of the Framework without compromising the coherence of the whole.
3. We must be able to build the Framework according to our ontology.

We selected a data structure and database manager which would help us to meet these requirements. The Framework is stored as a network within a graph database managed by *Neo4j*, an industry-standard graph database management system (GDBMS). CMF Nexus serves as a custom front-end for this graph database. The use of a graph database allows us to store, retrieve, and edit connections between mathematical ideas more easily than we would be able to if our data was in a flat table or a relational database. The front-end provides interfaces and tools for building queries to filter Framework content into meaningful subsets, and visualisations for viewing structure and content as maps, with details of map content available on demand. These are bundled together with Research Summaries, documents which tell the story of the structure and content of a subset with respect to the research base, all of which are written and viewed on CMF Nexus, and then sent out for on-line review. Research Summaries, maps and tables of content, and visualisations of content analysis facilitated by CMF Nexus are already playing a role in mathematics curriculum design within our pilot case studies.

Defining users, uses and user actions

Our overall problem context is the lack of coherence in students' (and teachers') experiences of mathematics due to differences between the curriculum as intended by designers, implemented in schools, and attained by students (McKenney, Nieveen, & van den Akker, 2006). Based on this context we identified our main overarching categories of potential Framework users and the general scopes of their work (see Table 6).

Table 6: Categories of potential users and uses of the Framework

Category of potential users	Category of uses	Scope of use
Curriculum committees	Curriculum goals, design, comparison and revision	Broad; multiple domains, entire range of content
Policymakers	Comparison and revision of current policies; policy advice	
Instructional designers	Designing or revising resources, activities, textbooks, schemes of work	Intermediate; switching between levels of detail for particular subsets of the Framework
Teacher educators	Designing and delivering teacher education programs; continuing professional development	Intermediate to detailed; switching between a horizon perspective, waypoints and student actions for very small subsets of the Framework
Teachers	Certification; professional development; lesson planning	
Assessment designers	Designing, developing or revising assessments to match the curriculum	
Researchers	Investigating existing literature regarding mathematics learning; developing new ideas and theories; enhancing current understanding	Detailed; working with themes, waypoints, student actions and research nodes for small subsets of the Framework

Other categories of potential users include assessment designers, researchers and policymakers. These categories are broad, overlapping and heterogeneous and would not be well represented in a single user profile. We have gained some clarity from a pilot series of interviews with representatives of user categories, described in Methodology: Formative evaluation. At the same time, we focus on identifying and supporting a set of core uses, some of which may be essential for all types of user and some of which may be essential but specific only to certain types. For example, all users may benefit from seeing where a topic fits across the network as a whole, but curriculum designers may need a different window onto that information than teachers would need.

This core set of uses is tested and added to in pilot cases. The general guiding questions shaping our approach to pilot cases are:

- What are the major uses and why/who? How does that translate into specific actions taken with CMF Nexus?
- What is it that only the authors of the content can do, vs. someone external with a high level of experience? A low level of experience?
- Which barriers to use can be reduced through interface design or data structure?

As we move towards user interface design, our guiding questions will become:

- What use cases should have the most influence on design?
- What core features are needed to make the Framework useful and meaningful for all user categories?
- What features are specific to the needs of particular groups of users?

In our pilot case studies we ask users to tell us how they have used the Framework and code their responses to identify:

- the specific actions they take with the Framework (e.g. filtering to a specific focus, changing focus, following the map forward or back, reading from context to detail, reading from detail to context, etc.), also shown in Figure 3, and
- the immediate purpose or goal of their actions (e.g. test assumptions, write a textbook chapter, map content to the Framework or to some other curriculum, learn how two sets of ideas might be connected).

This process is described further in Methodology: Formative evaluation.

Formative evaluation and feedback in the design cycle

Feedback drives refinement through design cycles. It is essential for refining the design of the Framework while our major design efforts are still in progress. We gather feedback from people representing potential users of the Framework and the research communities whose work we are drawing on in the literature as we develop Framework content. In this way we expand our circle of participants in design and incorporate essential perspectives into our knowledge building discussions. Our strategy and methods for formative evaluation of Framework design and content are described in Methodology: Formative evaluation.

Conclusion

From developing our design problem, goals and principles to building, evaluating and refining the Framework, each component of the design process described in this paper involves decision-making which is informed by a variety of considerations. We have explicitly named and described these, with the reasoning behind them, so that our work can be understood and interpreted in context. We also hope that providing this perspective will contribute to the resources other designers can draw on for similar projects. Other papers in our Methodology series[11] expand on some of the themes introduced here.

[11] Methodology: Building the research base, Methodology: Formative evaluation, and Methodology: Glossary app (Majewska, 2019)

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