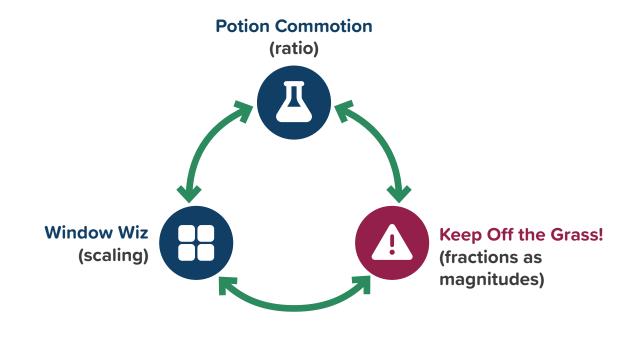


Teacher support materials for Keep Off the Grass!



Connections to other Ratio Riddles activities

This is one of three activities in Ratio Riddles. These activities complement one another and provide a useful set of experiences that can help students build strong and flexible foundations in proportional reasoning:





Game activity and learning

Students have an opportunity to work with fractions representing distances. Traditionally this can be done with number lines or bar models, including manipulatives, which help students to work with fractions as proportions of a whole – where the whole could represent a distance unit or other quantities. There are a variety of bar model manipulatives; Cuisenaire rods are one example.



Challenge

In this activity, colored rods, representing different fractions of a whole unit of distance, are used to solve a puzzle. How can players build paths across the protected grass and rescue stranded students without running out of materials? To rescue all the stranded students, they must use the fewest rods possible to get across the grass.



Connections

This activity connects ideas of equivalent fractions, composition and decomposition of fractions, and fraction addition. As they solve pieces of the puzzle, students will get feedback, and practice composing and decomposing fractions as distances as they consider which composition of equivalent fractions is best for reaching the final goal.



Scaffolding

Difficulty starts low and increases. First students only need to consider simple unit fractions they have probably encountered before, like $\frac{1}{2}$. Then they will also need to consider fractions with larger denominators, like $\frac{1}{12}$ or $\frac{1}{24}$ which they don't commonly encounter, as well as numerators greater than 1. Finally, they will need to compose fractions by combining measuring rods representing different unit fractions, like using $\frac{1}{3} + \frac{1}{4}$ to make $\frac{7}{12}$.



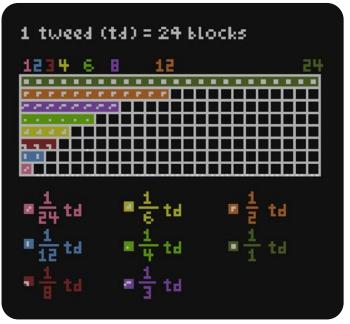


Gameplay guide

After starting the activity with the Dean of Students, the player follows the Professor of Cartography to a walled garden. A starting tutorial shows players the various lengths of measuring rods they can use, both in terms of blocks, the smallest unit of space in Minecraft, and in terms of a distance unit, where 24 blocks = 1 tweed (td).



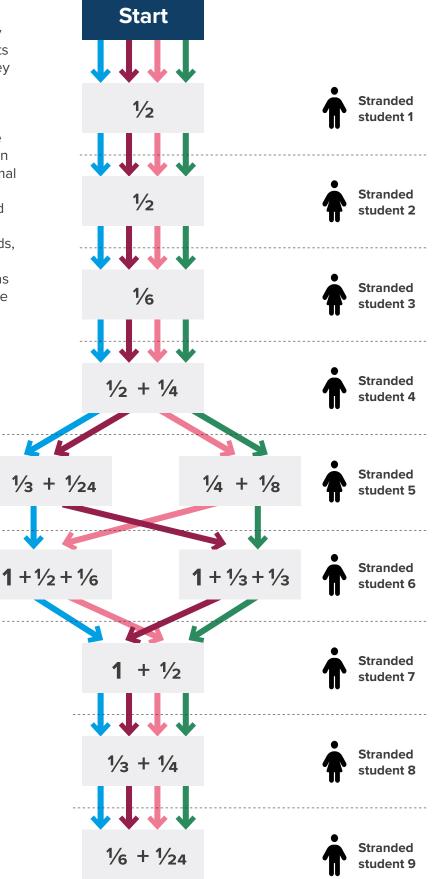
Each time players reach a stranded student, the Professor of Cartography will give them feedback about their measuring rod choices. They'll be told (a) if it was an exact solution to span the distance, and (b) if they should find a way to use fewer rods to compose the same fraction – in which case they'll be shown a diagram reviewing what they've tried with lots of positive feedback and appropriate framing.





This diagram shows all possible correct solutions at each step, and the four colour lines represent the four different (but equally correct) decision-making trajectories students may follow. Depending on which choices they make when saving students 5 and 6, they may have slightly different numbers of some rods in their inventory. Note that there are two possible combinations of rods which are both the 'fewest number' players could use in steps 5 and 6; either of these is a valid minimal solution the game will accept. Players are given enough rods to rescue all the stranded students if they use as few as possible for each step. If players run out of necessary rods, it means that at some point they chose to rescue a student without using as few rods as possible, and they will have received in-game feedback to inform them.

Once a player has saved four students, they will have the opportunity to stop there and go through Junior Graduation. If they save all nine students, they can choose to go through Senior Graduation. Once they graduate from the activity, the Dean of Students gives them a portfolio and a camera; they can take pictures anywhere in the activity and export their portfolio as a keepsake if they wish.





Concept focus

Fractions as distances

- Knowing that fractions can represent physical lengths (as well as proportions) is important. Placing fractions together on a number line can help students overcome natural number bias, where, for example, they might inappropriately add denominators when adding fractions because they consider the components of the fractions separately rather than together as an expression of one number.
- Working with unit fractions helps students to keep a consistent 'whole' in mind in measurement contexts. Doing this can contribute to learning to place fractions on a number line, which can help them to understand fractions as magnitudes and to move beyond whole/natural numbers to an early sense of rational numbers. It can also ground the meaning of non-unit fractions as multiples of unit fractions, which can help students to begin working with more complicated fractions.
- Working with multiple representations of fractions can support flexibility and depth of engagement.
- Measuring distances demonstrates the real-world applications of fractions as distances in navigation, and potentially also for building/architecture in the real world.

Key mathematical terms

- Length: The distance from one end of something to the other in a straight line.
- Fraction: A way to write the number which results from dividing one whole number by another. A whole number can be written as a fraction, like ³/₁, but they are often used to write numbers which are not whole, like ²/₃, 1⁵/₆ and ²⁰/₇.
- Whole number: Any positive number you might use to count things, and zero.

Observe gameplay and prompt thinking

You may observe signs of early exploration of, or more developed engagement with fractions as distances, even if a student hasn't fully solved the puzzle.

- **Early exploration:** Students make random but practical choices, placing whichever rods will let them cross to the next platform. They might benefit from having a conversation about units while looking at the key to rod lengths and fractions which they can see in the tutorial area or in their inventory, as shown in the image on page 3. In this case, a block is the smallest unit, and the larger distance unit is the *tweed* (td). The various measuring rods are different fractions of that larger unit.
- More developed engagement: Students plan before making a move. They are trying to cross as much grass as possible using the fewest measuring rods possible on a step-by-step basis. In order to do this, they should choose the rod representing the largest possible fraction of the distance unit which doesn't overshoot the target distance, but they may not always succeed. They might benefit from a conversation about equivalent fractions while looking at the key to rod lengths and fractions which they can see in the tutorial area or in their inventory, as shown in the image on page 3. What are two different ways they could make up ½ with the rods shown?



After/before/in between playing, you might like to:

- Use physical manipulatives (Cuisenaire or other types of colored rods) to compose various fractions and talk about differences between what they do in the game and what they are doing at their desks. For example, in the game, they explore fractions as distances, but at their desks they could also explore them as volumes.
- Try one of these dice games about creating fractions, comparing them and placing them on number lines. Each pair of students will need a 0–9 die, paper, and coloured pencils or pens.

Version 1 (True and equivalent?)

The image of two "empty" fractions with a greater-than-or-equal-to symbol between them should be drawn by each student to begin the game. One student rolls the die three times and both students choose where to place these numbers in three of the four blank boxes on **their own** image. They each then choose a number 0–9 to fill the fourth blank box.

Scoring:

1 point if their statement is true.

1 point if the two fractions are also equivalent (this could be checked with calculators); for example:



Note:

Expect questions and discussions to be generated around zero being a numerator or denominator.

Version 2

A pair of students take turns to roll a 0–9 die twice. Each student uses the numbers they rolled to generate a fraction, which they mark on a number line in 'their' colour. New fractions are added each turn, and the winner is the person who manages to get three of their fractions next to each other on the number line.



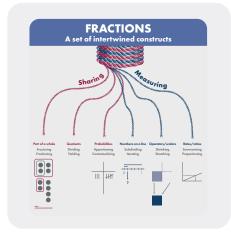




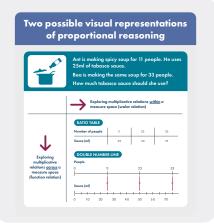


- Connect to NRICH tasks
 - » Age 5 to 7 Cuisenaire counting https://nrich.maths.org/problems/cuisenaire-counting
 - » Age 5 to 7 Same length trains <u>https://nrich.maths.org/problems/same-length-trains</u>
 - » Age 7 to 11 Linked chains https://nrich.maths.org/problems/linked-chains
 - » Age 7 to 11 Fractional wall https://nrich.maths.org/problems/fractional-wall
 - » Age 7 to 14 Rod fractions https://nrich.maths.org/problems/rod-fractions

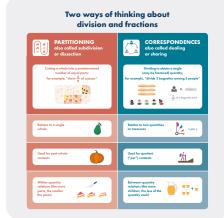
Further reading from Cambridge Mathematics



Espresso 25 – What does research suggest about effective ways to introduce fractions?



Espresso 28 – What does research suggest about the development of proportional reasoning in mathematics learning?



Espresso 40 – What does research suggest about the teaching and learning of fraction equivalence?





JourneyMaths – Sign up for a free account and view:

Instant: Exploring Fraction Equivalence

Instant: Establishing proportional reasoning

Visit JourneyMaths



Curriculum connection examples

The following are examples of curriculum standards to which this activity could contribute:

Common Core Mathematics Standards (United States)

- CCSS.Math.Content.3.NF.A.2.B "Represent a fraction a/b on a number line diagram by marking off a length 1/b from 0. Recognize that the resulting interval has size a/b and that its endpoint locates the number a/b on the number line."
- CCSS.Math.Content.3.NF.A.3.A "Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line."
- CCSS.Math.Content.4.NF.B.3.B "Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. Examples: ³/₈ = ¹/₈ + ¹/₈ + ¹/₈ ; ³/₈ = ¹/₈ + ²/₈ ; 2¹/₈ = 1 + 1 + ¹/₈ = ⁸/₈ + ⁸/₈ + ¹/₈.
- CCSS.Math.Content.4.NF.B.3.C "Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction."

National Curriculum in England

"Pupils should be taught to":

- Lower Key Stage 2, Year 3 Number fractions: "recognise and use fractions as numbers: unit fractions and non-unit fractions with small denominators"
- Lower Key Stage 2, Year 3 Number fractions: "recognise and show, using diagrams, equivalent fractions with small denominators"
- Lower Key Stage 2, Year 3 Number measurement: "measure, compare, add and subtract: lengths (m/cm/mm)"
- Lower Key Stage 2, Year 4 Number measurement: "convert between different units of measure"
- Lower Key Stage 2, Year 4 Number fractions (including decimals): "solve simple measure and money problems involving fractions and decimals to 2 decimal places"
- Upper Key Stage 2, Year 5 Number fractions (including decimals and percentages): "identify, name and write equivalent fractions of a given fraction, represented visually, including tenths and hundredths"
- Upper Key Stage 2, Year 5 Number fractions (including decimals and percentages): "recognise mixed numbers and improper fractions and convert from one form to the other and write mathematical statements > 1 as a mixed number [for example, $\frac{2}{5} + \frac{4}{5} = \frac{6}{5} = \frac{11}{5}$]"



Acknowledgements

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Credits

Learning design lead: Ellen Jameson Pedagogical design lead: Frances Watson Graphic design: Ray Knight Editing and proofreading: Lynn Fortin