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Roles and limits of curriculum frameworks in mathematics education

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"As long as learning math counts as learning to think, the fortunes of any math curriculum will almost certainly be closely tied to claims about what constitutes rigorous thought — and who gets to decide (Phillips, 2015a)."

Many people recognize and appreciate the fundamental contributions the field of mathematics has made to human culture, but see it as a one way street; practices in mathematics as a discipline are not popularly viewed as being socially dependent. The mathematics curriculum is another story. In various times and places in recent history, the mathematics curriculum has been enlisted in political fights against communism and capitalism, conformity and rebelliousness, discipline and laxity. On global, societal, and individual scales, mathematics education is perceived to influence, and be influenced by, social and economic power (Phillips, 2015b; Borovik, 2014). Where might the Cambridge Mathematics curriculum framework sit in this landscape? To what extent can a mathematics framework affect students' preparation to engage with the domain, the workforce, and the wider world? I will explore these questions with respect to three commonly held ideals in curriculum development: equity, neutrality, and evidence.

Equity

Ideas about what constitutes equity in mathematics education are fundamentally connected to what people want for society, each other, and themselves. According to the National Council of Teachers of Mathematics (NCTM), achieving equity involves providing students with access to good curriculum and instruction, differentiation for productive engagement, sufficient time for students to learn, and the strategic use of resources. If strategies for upholding equity are successful, then demographic differences among students shouldn't be predictive of student outcomes (NCTM, 2014). Strategies for achieving and upholding equity require support on multiple scales and depend on system-wide and local decisions in the political arena.

Policy negotiations cannot be completely free from bias, even for policies designed to promote equity through a neutral approach (see 'Neutrality' below). This has implications for the promotion of equity in

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education. In The Politics of Official Knowledge (1993), Michael Apple asserts that "[e]ducation is deeply implicated in the politics of culture. The curriculum is never simply a neutral assemblage of knowledge…It is always part of a selective tradition, someone's selection, some group's vision of legitimate knowledge (p. 222)." Different jurisdictions may have very different strategies for managing who has the power to make curriculum decisions, but in the interest of serving society and successfully enacting policy, neutral language is often used to portray policy actions in support of curricular aims as objectively optimal in their intent on behalf of all (Apple, 2004). Policy makers may use evidence from educational research to evaluate the means of achieving educational goals, but also to strengthen claims of relative neutrality regarding equity in policy decisions (Walshaw, 2010).

Schools link the fates of individual students and whole communities to the designs of powerful groups (Apple, 2004), which may be better suited to some students than others. Policy makers, in creating policy that governs the education system, are making political decisions affecting all of their constituents, and the less power constituencies have, the less influence they tend to exert in political decision-making (Levin, 2008). While leaders in every jurisdiction want to educate students, the problem with assuming that this education will unambiguously optimise all students' opportunities is that some of the goals for the structure of educational systems involve maintaining the existing knowledge hierarchy, which is also tied to the existing socioeconomic hierarchy through cultural norms and the labour market (Apple, 2004; Borovik, 2014).

Through school systems and enacted curricula, we risk reproducing inequality by perpetuating existing advantage and disadvantage, in mathematics and all areas of the curriculum (Johnson, 1991, in Apple, 1993; Apple, 2004; Bourdieu, 1974). Bourdieu's concept of cultural capital describes the implicit benefits of legitimate knowledge that members of dominant cultural groups gain and express through fluency with the dominant language and culture (Bourdieu, 1974). Often, these benefits contribute to better performance in a school setting (Apple, 2004; Sullivan, 2001). Although it is now more generally acknowledged that students do not all enter school on a level playing field, it is still unclear where the boundaries lie (Levin, 2008) – what implicit knowledge must disadvantaged students gain to "catch up," how can schools help them cross those lines, and how will we know? Those who achieve fluency with this knowledge through study rather than enculturation may still not be accorded the same benefits as those who seem to be naturals (Apple, 2004; Bourdieu, 1974), and students who do not perform well may be judged as if their performance reflects only their ability.

In spite of this, schools are also in a position to support the foundations for greater social equality by fostering what Ernest (2002) calls "critical mathematical citizenship" (see Figure 1). This idea includes broad preparation in numeracy and data literacy, elements of the distribution of skills that would enable an informed citizenry to evaluate evidence in the decision-making process. Such beliefs about the benefits of mathematics education for citizenship have grown and changed since the mid-20th century, when the study of mathematics was broadly believed in Western countries to enhance cognition in proportion to the amount and depth of study (Phillips, 2015b). During the New Math movement in the 1960s, citizenship goals were focused more on widening the pool from which expert practitioners could be drawn to fill expanding roles for engineers, scientists, and mathematicians (Phillips, 2015b). At that time, the labour market also supported a wider range of medium-skilled jobs, which made it easier to argue that higher levels of mathematical preparation were likely to be useful for a large proportion of students (Borovik, 2014).

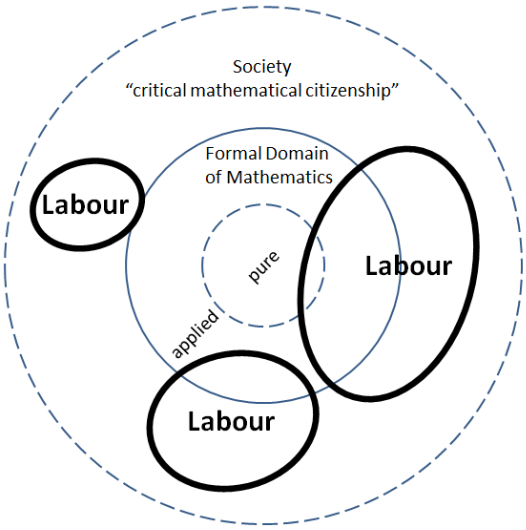

Figure 1: This is one diagram of many that could be drawn to illustrate the integration of mathematics throughout society. The intersection of various labour pools with the domain of mathematics ranges from extensive to scant.

Today, school systems can elect to support equal access to the labour pool by preparing more students to compete for high-status jobs requiring advanced mathematics, but because of the widening split in the job market's technological skill requirements, many jobs either require much more or much less preparation in mathematics than the median (Borovik, 2014). Now, as in the past, mathematics credentials are particularly strong and widely utilised gatekeepers for specialised opportunities (Ernest, 2002), but they contribute to preparedness for work in a lower proportion of jobs than they once did. In consequence, society's leadership faces pressure not to invest in universal public education in mathematics beyond the lowest common denominator, which is lower now than it has been since the introduction of universal education (Borovik, 2014).

Because reinforcement and development of certain mathematical content throughout early education prepares the way for more advanced work later, it is difficult to manage the transition from concrete to abstract mathematics (Key Stage 3 in the UK) successfully for students who will go on to do more advanced work without laying the groundwork for all students beforehand (Borovik, 2014). Furthermore, when students reach that transition point, the labour market may no longer provide a compelling reason for many of them to push through it. As a result, the supply of future mathematics teachers (needed to educate even a small number of experts) could be in danger of dwindling, and the number of students who will have developed the experience necessary for critical mathematical citizenship will be limited.

In spite of these drivers, and whether or not any part of the mathematics domain can truly be considered culturally independent, there is wide agreement about many key dimensions of the domain of mathematics across cultures. However, the extent to which these are culturally expressed, or dependent on culturally interpreted signs and symbols, is an open question (Apple, 2004). Even within a single culture over time, as with New Math, presenting essentially the same topics, order, and configuration but from a different point of view on the utility of maths had (and has) implications for the way students learned and how well the effort was integrated into existing educational institutions (Phillips, 2015b). Cambridge Maths can't control for all factors affecting equity in mathematics education, but we seek to provide the flexibility for jurisdictions to use the framework as effectively as possible according to their goals for all students.

Evidence

Those responsible for setting educational goals may turn to tradition or evidence when seeking to revise curriculum and practice. Tradition carries the advantage of being known to have achieved certain results for a proportion of students in the past, but presumably goals are being updated in order to achieve a different result from outcomes under the current system. Therefore, policy-makers and curriculum designers turn to evidence from educational research to address the problems they wish to solve. Applying evidence from the classroom is becoming more appealing to policy makers and funders because it implies a productive cycle of feedback between teachers, students, and officials (Walshaw, 2010). However, it means that the most influential forms of evidence, such as standardised assessments, can be strong drivers of educational practice among teachers (Black, 2012, in Horsman, in press) and habits of mind among students (Denizhan, 2014), which might artificially restrict the range of useful experiences available to students.

There are cognitive, behavioural, and social dimensions to learning in classrooms. Educational research can focus on one or more of these in settings that range from testing or laboratory environments to completely naturalistic classroom environments. Every kind of classroom research involves trade-offs between capturing effects that emerge from meaning-making and motivation in a complex learning environment and the ability to measure and interpret learning outcomes.

On the most rigorous end of the spectrum, randomised controlled trials and longitudinal studies measure outcomes achieved within particular learning environments. Conceptual analysis can map out a detailed landscape of students' ways of knowing conceptual elements within a given topic in mathematics. Only a small portion of the curriculum has been taken up in conceptual analysis studies, and they are resource-intensive. Randomised controlled trials are also resource-intensive, and raise questions of validity in naturalistic settings since it is often not possible to control for many potentially relevant variables at the classroom activity level. Longitudinal studies are subject to the same problems as randomized controlled trials, with repetition and attrition adding to the burden of resources required.

Currently, the evidence that carries the most weight in evidenced-based curriculum planning comes from standardised tests and empirical studies involving randomised controlled trials (Walshaw, 2010). However, the NCTM Research Advisory Committee, 2003, noted that restricting the evidence that is used to inform mathematics teaching and learning to these sources may bypass equally essential insights that can only be obtained through other research designs. Traditional research design, while borrowing authority from its role in medicine, psychology, and the natural sciences, "excludes non-cognitive and social student outcomes and tends to prevent important local studies of diverse communities to surface. What are concealed are the critical realities of different contexts, policies, systems, resources, approaches, and practices as well as the different ways in which they impact on students (Walshaw, 2010 p. 17)."

The community of educational researchers has been increasingly accepting of the use of qualitative approaches from the social sciences, both to assist in interpreting quantitative approaches from cognitive psychology and to yield new, complementary insights (Frade et al., 2013). To further our goal of basing the Cambridge Mathematics framework upon the evidence that is currently best able to describe effective mathematics teaching and learning at a sufficient level of detail, we are working with educational researchers from a variety of backgrounds to provide diverse research perspectives while maintaining a high benchmark for the quality of research influencing the discussion.

Neutrality

What students study when they study mathematics, anywhere in the world, is a subset of the domain which is selected by various balances of stakeholders in the curriculum development process (Apple, 2004). Beyond those initial choices, the way students study mathematics is determined by much more than everyone's ideas of the domain itself; more, even than theories of teaching and learning and curriculum design. No stage of curriculum development and implementation is free of the judgements and preferences of stakeholders with a range of professional goals (see 'Equity' above).

Accordingly, no framework is completely neutral. By nature, a framework is a construct we build to make useful sense of the infinite detail and nuance of reality, and what is useful depends on context. We make decisions about structures and features in order to highlight what is considered most important; this differentiates the framework-construct from the endless detail of reality. However, in the case of a mathematics curriculum framework, we believe it is possible to make the framework as applicable as possible to a variety of goals and contexts by including a balance of international perspectives through research and personal consultation. Individual jurisdictions have to make their final curricula work in their cultural, political, and economic contexts and may choose very different investments and trade-offs, but many conceptual elements are still held in common and decisions around these can benefit from the Cambridge Mathematics framework.

Context also matters in mathematics education below the level of wider cultures and jurisdictions. Categorically, we are working on a curriculum framework for schooling in some type of formal system, whether in schools or at home. We therefore work within a context that carries with it a host of issues related to formal schooling, only some of which can be directly influenced by students' mathematical experiences. Having assumed that context, we seek to provide a basis for curriculum designers and teachers to build good learning trajectories, even as we fully recognise that some ideals of mathematics education may not be approachable within these structures as they currently stand.

Conclusions

Any mathematics framework underlying curriculum development will be one of many factors influencing the experiences students have with mathematics throughout their educations. The way in which the framework is adopted into curricula, taking into account goals and contexts specific to particular jurisdictions, matters as much as the framework itself. However, the details of the framework do have the potential to make a positive difference in students' experience with mathematics in school and beyond. Cambridge Mathematics is working to build a framework that incorporates evidence from a high-quality research conducted according to diverse and complementary perspectives and employing a range of appropriate methodologies. We recognise that we cannot provide a substantially different context for implementation than other frameworks face, but we hope that we can build a solid foundation for the development of mathematics curricula around the world which enable desired levels of engagement with the domain of mathematics, provide students with the skills and understanding they need in the workplace, and support the growth of critical mathematical citizenship.

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