Use of Money as a Decimal Representation: A Review

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Research addressing the strengths and weaknesses of using money as a representation of decimal concepts falls within the general categories of research on teaching rational numbers, fractions, and decimals. This research is linked to the literature examining strengths and weaknesses of real-world problem solving, perceptual richness in contextualized activities, misconceptions about decimals, and forms of representation of decimal concepts. Within this body of literature, money is most often involved tacitly in problem-solving and assessment, without comment on its particular affordances as a representation. Work that does address money as a representation directly has been done in two connected areas: (1) money, symbolisation, and language, and (2) money and manipulatives. Theory and empirical findings in both areas suggest a few key strengths and weaknesses of money as a representation of decimal concepts which would be important to take into account for the framework.

Summary of advantages and disadvantages

Advantages	Disadvantages
• In theory, counting money could help students with decomposition of numbers to the value of their digits • Can help students leverage informal knowledge • Fewer conceptual errors were made by students using perceptually rich money • Might be assets in activities placing a greater emphasis on problem-solving than accuracy	• Two noted misconceptions associated with money as a symbolic representation (longer-is-larger or shorter-is-larger) • Money as a concrete representation is associated with one of these misconceptions • Money as a perceptually rich concrete representation can adopt the disadvantages of perceptually rich manipulatives (see below) • Thinking and talking about money using everyday language, whether in conjunction with manipulatives or symbols, can also reinforce one of the above misconceptions

Money in language and symbolic representation

The processes variously described as gradual decontextualisation (Gravemeijer, 2002), progressive symbolisation (Enyedy, 2005), and concreteness fading (McNeil & Fyfe, 2012), widely used in the teaching of mathematical concepts in general, including decimals, lie within Freudenthal's concept of horizontal mathematisation (1991). "In his last book Freudenthal (1991) adopted Treffers' distinction of these two ways of mathematizing, and expressed their meanings as follows: to mathematize horizontally means to go from the world of life to the world of symbols; and to mathematize vertically means to move within the world of symbols." He found the two forms of mathematizing to be of equal value, and stressed the fact that both activities could take place on all levels of mathematical activity. (van den Heuvel-Panhuizen, 2003). Using money as a representation of decimals is neither abstract nor a completely concrete representation of the concept of decimals. While it therefore has the potential to help students with horizontal mathematisation, it also carries particular associations with language that can complicate its use in the classroom.

Disadvantages

Steinle (2004) reports the major findings of a longitudinal study of understanding of decimal notation as expressed by decimal ordering. The study roughly categorized students' performance according to whether it demonstrated procedural expertise, one of two common misconceptions (longer-is-larger or shorter-is-larger), or other errors, and identified ten categories of misconceptions within the three broad error-displaying categories. Steinle linked one of these categories, *money thinking*, to money directly, while another, *string-length thinking,* was not linked to money in this study but closely resembles misconceptions around money and decimals noted by other researchers:

- Money thinking: students are able to order decimals correctly up to two decimal places but then fail to correctly order numbers which only differ beyond two decimal places corresponding to cents (Steinle, 2004).
- String-length thinking: students "interpret the decimal as two whole numbers separated by a marker (Steinle, 2004, p. 2). This misconception is similar to an issue noted by other researchers.

Brekke notes an issue similar to string-length thinking:

"[t]eachers regularly claim that their pupils manage to solve arithmetic problems involving decimals correctly if money is introduced as a context to such problems. Thus they fail to see that the children do not understand decimal numbers in such cases, but rather that such understanding is not needed; it is possible to continue to work as if the numbers are whole, and change one hundred pence to one pound if necessary. It is doubtful whether a continued reference to money will be helpful, when it comes to developing understanding of decimal numbers; on the contrary, this can be a hindrance to the development of a robust decimal concept." (Brekke, 1996, p.138, in Steinle, 2004).

Language and the string-length thinking misconception

In a report published by Babcock LDP concerning language and the use of money in talking about decimals, the authors point out that students often treat pounds and pence (whole number places and decimal places) as if they are separate whole numbers rather than a single decimal number, and that this is due at least in part to the use of whole-number language when describing decimal amounts of money (e.g. "three pounds and seventy-six pence"). Using measure as a representation in decimal problems can provoke the same type of error if the same language construct is used (e.g. "2m and 30cm" instead of 230cm)(Babcock LDP, accessed 2016). NCTM working group participants acknowledged this in a circular, raising the possibility that vocal reading of money using everyday language could be problematic (NCTM, 2003). Steinle also notes that "students (and textbooks) often don't distinguish between the digits (0 to 9) and the numbers created from various combinations of digits. We need to use clear language in the classroom which will help students to know when we are focussing on either the "forest" or a particular "tree"." (Steinle, 2004, p.7).

Advantages

Carraher et al. (1988) point out that counting money involves both the absolute value of the number of coins or bills and the relative value of the coins and bills on a single scale. They theorize that this might be able to assist students in decomposition of numbers to the value of their constituent digits. In 1985, Carraher conducted well-known studies demonstrating that adults and children who couldn't write numbers but had experience using money could successfully perform arithmetic calculations to solve problems in the context of money. In one study, "children were 98 percent correct in their street calculations but only 37 percent correct in their school calculations, even though the same numbers were used" (Carraher et al., 1988, p. 42). However, as other researchers have mentioned, decimal understanding is not always necessary for successfully solving money problems (Brekke, 1996, in Steinle, 2004), and students who can correctly do money problems involving two decimal places do not necessarily demonstrate the ability to correctly deal with three or more decimal places (Steinle, 2004; Babcock LDP, accessed 2016).

There are few studies that have directly compared money to other decimal representations on an assessment. One is a small action research case study facilitated by an educational researcher (Martinie & Bay-Williams, 2003). This is a small study of 43 students age 11-12 and one teacher and includes no statistical analysis of assessment results (comparing responses to random guessing, for example). Students completed an assessment consisting of problems requiring the use of four different representations of decimal numbers: number line, grid, money, and place value. Students' answers were 65% correct when given in terms of money and a 10x10 grid, 58% correct when given in terms of place value, and 26% correct when using a number line. These results support further exploration of the idea that using money as a decimal representation can help students to draw on their informal knowledge, but this study does not address the potential shortcomings, as the assessment may not have created similar to Steinle's (2004) design in which those shortcomings might have become apparent.

A similarly small study (16 students age 11-13 from a single classroom) investigated how students leverage everyday understanding to build a more formal understanding of decimals. This study included money problems in its assessment (Irwin, 2001). In a comparison of work on contextualised and non-contextualised problems, Irwin found that students who worked on contextualised problems, including some involving money to three or more decimal places, performed better on an initial assessment and demonstrated greater retention on a follow-up assessment two months later. This finding is consistent with Carbonneau et al.'s (2013) findings on improved retention through the use of concrete manipulatives, although we are continuing to review the literature on contextualised problem-solving and retention to explore whether there is a sufficient theoretical basis for including that in our consideration of money and decimal representations.

Advantages and disadvantages

Money can be used as a manipulative in either a perceptually rich (more detailed and contextualised) or a bland way (less detail may allow for more general application). Research reviewed by Carbonneau et al. (2013) suggests that perceptual richness "may hinder learning of targeted mathematics concepts and/or performance solving mathematics problems (Kaminski, Sloutsky, & Heckler, 2009; McNeil et al., 2009; in Carbonneau et al., 2013, p. 381)." Carbonneau et al. report three primary risks in using perceptually rich manipulatives (this list is not specific to decimals or money):

1. Risk of inadvertently providing irrelevant or misleading cues that divert attention from the target concept (Martin, 2009)
2. Risk that concrete experience will dominate completely and students will rule out an abstract connection (Kaminski et al., 2009; McNeil et al., 2009)
3. Risk that even if an abstract connection is made, students may not use it to successfully apply the target concept in other contexts (Uttal, O'Doherty, Newland, Hand, & DeLoache, 2009)

McNeil et al. (2009) conducted a randomised controlled trial examining errors made by students age 9 - 12 years in word problems about operations involving money. Students worked with decimals in the context of money using either perceptually rich money (visually similar to real money or play money), bland money (black and white rectangles for bills and circles for coins, with a value printed in the centre), or no manipulatives at all. The students using perceptually rich money manipulatives made more errors overall, but fewer of these were conceptual errors compared to the bland manipulative or control conditions.

Both the meta-analysis (Carbonneau et al., 2013) on manipulatives in general and the study by McNeil et al. (2009) on money in particular posit tradeoffs from the use of perceptually rich manipulatives, but these are somewhat contradictory. The more general meta-analysis concluded that high perceptual richness significantly benefited transfer outcomes, but was marginally detrimental to problem-solving and was not as effective for retention as low perceptual richness. The study specifically targeting money concluded that perceptually rich money manipulatives may help students to leverage their informal knowledge, but may also hinder formalization of that knowledge and the application of knowledge in a different context.

Recommendations from research

- Perceptually rich money might be helpful in activities with a greater focus on problem-solving, but might be counterproductive in activities depending on greater accuracy, like summative tests (McNeil et al., 2009).

- A process of gradual decontextualisation might help bring students closer to realising the essential ideas behind the particular problems they're solving (Gravemeijer, 2002, in McNeil et al., 2009).
- NCTM emphasizes vocal reading to reinforce the meaning of each place and the distinction and connection between the right and left sides of the decimal point – but specifically mentions the vocal reading of currency as an exception, since this could instead reinforce misunderstandings (NCTM, 2003 - unpublished working group circular) like those discussed above. This problem could potentially be avoided by reinforcing standard decimal vocal reading of currency notation (e.g. "two point five four pounds" rather than "two pounds fifty-four p."). Ball, (1993) emphasizes that it is important to carefully develop the context for classroom discourse in order to make money work as a representation.

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