SHARED PERSPECTIVES ON RESEARCH IN CURRICULUM REFORM: DESIGNING THE CAMBRIDGE MATHEMATICS FRAMEWORK

Ellen Jameson, Lynne McClure, Tabitha Gould

Cambridge Mathematics, University of Cambridge

Different groups of stakeholders in curriculum development hold different perspectives on teaching, learning, and the domain of mathematics. The degree to which they coordinate or align their efforts based on these perspectives affects curriculum coherence, both in the design of the intended curriculum and how it is enacted in schools. Stakeholders must be able to share or at least understand each others' perspectives on the potential implications of research for curriculum design in order for research to have a coherent influence. We are designing the Cambridge Mathematics Framework to link research to mathematics learning in a form that can be mutually considered and applied to the processes of curriculum design and enactment by curriculum designers, resource designers, and teachers. We describe work in progress on the design of the Framework and the processes underway to incorporate feedback into the design and evaluate whether the Framework represents research in such a way that it is likely to be meaningful, useful and used.

INTRODUCTION

The decisions and actions of a diverse set of stakeholder groups shape the ways in which any given mathematics curriculum is *intended* to function by its designers, *enacted* in schools, and *received* by students (Stein, Remillard, & Smith, 2007). Coordination, or lack thereof, between these groups affects how coherently the domain of mathematics is presented in the intended curriculum, and how coherently the intended curriculum can be enacted. These each affect what mathematics students have the opportunity to learn (Schmidt, Wang, & McKnight, 2005). Research in mathematics education – including philosophy of mathematics learning, learning in particular subdomain areas, and pedagogy – has the potential to be applied in the design and enactment of curricula. However, different stakeholder groups (and different stakeholders within groups) are likely to be familiar with different subsets of existing relevant research, and when they look at the same research they don't always see it in the same way. This might limit the effectiveness of actions that any one stakeholder might take based on this research, if these actions are not coherently supported by the work all groups do to form the curriculum as a whole. In this paper we describe work in progress on the design and evaluation of the Cambridge Mathematics Framework, which we intend will help to coordinate perspectives on applying research in curriculum design for three umbrella categories of stakeholder roles: curriculum designers, resource developers, and teachers. Of the processes of design, development, and reform that drive curriculum change, our project is focused on contributing to curriculum design and development, but we work with the larger process of curriculum reform in mind.

Curriculum coherence and a shared perspective on existing research

'Coherence' is frequently called for across the curriculum design and mathematics education literature as a way to increase effectiveness of teaching and learning, by coordinating policies, resources, and actions. Discussions of coherence in curriculum reform often fall into two categories: cultural and cognitive, each with a distinct set of implications for the goals and design of the Framework.

A cultural lens focuses on curriculum coherence through coordination of diverse perspectives (Hall, Morley, & Chen, 2005; Robutti et al., 2016; Thurston, 1990) or standardisation towards one perspective (Pring, 2012; Schmidt et al., 2005). This affects how a curriculum is decided upon and enacted through the education system. These cultural approaches need not be mutually exclusive but can be invoked depending on the nature of a given curriculum change (Schmidt et al., 2005).

A cognitive lens places the focus on coherence in the learning process that a curriculum is intended to support (Cobb, 1988), or the nature of the domain of mathematics itself (Dewey, 1938; Schmidt et al., 2005; Thurston, 1990), abstracted from individual experiences. In each context, the scale at which coherence is discussed can range from single concepts and individual learners through to regions and entire jurisdictions. The implications of aiming to support coherence, both for equity and for the structure of the curriculum, may therefore be very different.

We apply these two perspectives on coherence to our design in different ways. From a cognitive perspective, we represent mathematics learning according to a particular set of considerations for *what* is described and *why*, and *how* it can be experienced by students through their actions. We share this representation with members of the communities of practice that generate, review, and improve the research we refer to in our research base. From a cultural perspective, we seek to support curriculum coherence by designing the Framework to present research in a form that is relevant to stakeholders when they are making decisions. In this way we hope the Framework will help to foster shared meanings and practices in communities with diverse perspectives. Shared knowledge representations have been shown to facilitate working between groups who have differences in their constraints and priorities (DiSalvo & DiSalvo, 2014; Lee, 2005; Robutti et al., 2016; Star & Griesemer, 1989). We can't control how widely across a system the Framework might be used, but we can seek to make it appropriate for use in coordinating curriculum approaches, materials, and actions.

Designing and evaluating the Cambridge Mathematics Framework

When released, the Framework will comprise (1) a database of mathematical ideas and experiences, defined, referenced, and exemplified as actions and informed by research synthesis and consultation, (2) an interface providing a set of tools for searching and visualising mathematical content and the research base, and (3) a guiding structure that determines what and how ideas are expressed in the database. Eventually we also plan to include connections to specific classroom activities, assessments and professional development resources.

Our design process, described in the methodology section, is guided by the following questions: how can the contents of a mathematics curriculum framework be expressed in a way that: (1) has core features that designers, teachers, and subject experts can interpret and assess relative to their context? (2) emphasises connections? (3) expresses and describes research influences in localised parts of the framework and across the structure of the framework?

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Our evaluation of the design is likewise guided by the following questions: is this framework (1) as informed and meaningful as we can make it given the resources at our disposal? and (2) does it make reasonable use of existing research and feedback from collaboration and evaluation?

METHODOLOGY

We consider our approach to be research-informed design. It is a qualitative, interpretive process of expressing mathematics learning, combining theory and empirical research from a variety of sources with descriptions and experience from practice in a way that is explained and documented at a fine-grained level. We draw on models for design processes in education that have been developed and refined within design research methodology in education for over twenty years (Cobb, Confrey, diSessa, Lehrer, & Schauble, 2003; McKenney & Reeves, 2012). Particular aspects of design research that make some of its methods appropriate for us include: linking specific design priorities and choices to theory; using initial design work to develop design principles that inform ongoing work; engaging in iterative cycles of design in which feedback on work in progress is incorporated into new design versions and practices; and participation in design by experts in multiple relevant communities (Barab & Squire, 2004; McKenney & Reeves, 2012; van den Akker, Gravemeijer, McKenney, & Nieveen, 2006). Although our goals differ in some important respects from the goals of design research, we aim to conduct and document our work in such a way that our resulting design might later be able to contribute to research. However, until the Framework is complete enough to be implemented, we rely on face validation with experts to evaluate the content and the structure of the Framework. We are aware that such validation may not lead to generalisable conclusions with respect to curriculum design.

Comparable approaches have been described by framework development projects in other contexts. In a retrospective review of the standards writing process for the NCTM *Principles and Standards of School Mathematics* framework, the writers noted that a set of theoretical perspectives emerged as important influences over time as they collected, analysed, and incorporated feedback on work in progress (Ferrini-Mundy & Martin, 2003). Currently, the UNESCO Institute for Statistics is developing the Reference List & Coding Scheme (RL&CS) framework, intended to provide rich qualitative support for mapping theory and curricula to assessment frameworks. This project's interpretive approach similarly required the designers to evaluate trustworthiness on the basis of practical value of the construct, as expressed through feedback on work in progress by expert members of the communities that would be making use of it (Cunningham, 2017).

Literature review and the research base

We want research to influence the design and contents of the Framework in a way that is meaningful and valid. As mathematics curriculum framework designers have noted in similar contexts, however, there is more to draw on in the literature for some areas of mathematics education than others, and it is also more feasible to employ review methods that identify relevant, essential areas and themes than to complete exhaustive systematic reviews of work in every subdomain (Cunningham, 2017; Ferrini-Mundy & Martin, 2003; Sfard, 2003; Thomas & Harden, 2008). This means that while a design can be grounded in research it cannot be *prescribed* by research, and we do not suggest that our design is the only way of interpreting the research. Rather, we want our design to draw on existing research in the context of our goals for the Framework as a whole. These include considerations that are as much

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about how mathematical ideas are represented, recognised, and put into action by different users as they are about the nature of concepts, skills, practices, etc., that have been identified and characterised in mathematics learning.

Our inclusion criteria for this process are broad and functional, with the understanding that use of sources will be subject to further review as part of our external expert review process (described below). We note and exclude any source from a particular review that we judge to be irrelevant or lacking reasonable support for its claims. To facilitate expert review and our goal of transparency, we record metadata for research sources to help characterise and communicate our influences. This includes: (1) the source's level of influence on a particular area of the Framework, (2) the search method that retrieved it, (3) publication context and intended audience, and (4) broad category of focus. When sources are used in writing Framework content, they are entered into the Framework database and linked to that content. This makes it possible for writers, reviewers, and users to summarise and examine the influences that have contributed to specific areas of the Framework.

Design and evaluation processes

We have developed a guiding structure for positioning ideas in the Framework that allows us to make them explicit, set scope and boundaries, and find patterns. In this way, it acts as an ontology (Schneider, Siller, & Fuchs, 2011), which Gruber (1993) defines as "the objects, concepts, and other entities that are presumed to exist in some area of interest and the relationships that hold among them." This ontology is not fixed but is something we are continuing to add to and refine. Like any model, our ontology highlights or includes some ideas at the expense of others – often by intent, but sometimes as an unintended consequence of another decision. This means that our Framework may alleviate some problems involving shared understanding while failing to address others, and it is essential that we evaluate our decisions and their implications so that we can both communicate them and identify important changes to make.

We treat designing the ontology and writing the contents of the Framework as intertwined processes. We laid the groundwork for the design with high-level review of theories and approaches and we continue with cycles of review, writing and refinement. Initially, we reviewed a variety of perspectives on classification schemes and 'big ideas' in mathematics education, as well as curriculum frameworks and content documents from a selection of jurisdictions. We used ideas from this process to create a tentative "top-down" way of dividing parts of the domain among members of the writing team. At the same time, we imagined what we might need from the construct from the bottom up and reviewed existing frameworks for conceptual understanding in mathematics (Freudenthal, 1983; Michener, 1978; Pirie & Kieren, 1994; Schoenfeld, 1992; Skemp, 1979; Tall, 1988, 1999; Usiskin, 2015; Vergnaud, 1996 among others) and for learning with understanding (Bransford et al., 2000; Hiebert & Carpenter, 1992; Kieran, Doorman, & Ohtani, 2015; Martin A. Simon, Nicora Placa, & Arnon Avitzur, 2016; Sfard, 2003; Simon & Tzur, 2004; Swan, 2014).

Currently we are in a cycle of writing, discussion, feedback, and refinement of the content and the construct. We have developed a set of tools for writing content into the structure of the Framework, searching and visualising content, and collecting reviews of content. Each writer works according to a cycle of (1) literature review, (2) generation of content, relationships, glossary definitions, research records and any other features called for in a particular area (discussed in more detail below), (3)

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internal discussion and review, (4) formal and informal external evaluation, and (5) refinement based on feedback.

Good feedback is necessary in order for this cycle to be effective. While informal review has been ongoing, we have enough work in place to begin more formal evaluation, which we divide into processes for two aspects: expert face validation of the structure of the Framework in general (ontology), and the representation of mathematical ideas in specific topic areas. To evaluate the Framework ontology, we are currently conducting a Delphi study with a panel of experts in mathematics curriculum research and curriculum design. Delphi is a structured group survey method for identifying areas of consensus and dispute among experts (Clayton, 1997). It is especially useful for ontology evaluation because it allows us to work with a range of international experts who could not otherwise be convened in the same place, and it helps to mitigate some forms of bias in face-to-face interactions between members of specialised communities. We expect to be able to report the results from this Delphi study in late 2018. When evaluating specific topic areas, external reviewers will be provided with access to the visualisation and search tools used by the writing team. We will then gather feedback through surveys and semi-structured interviews.

At the same time, we are working to characterise relevant existing ways of working among potential users of the Framework so that we can anticipate discrepancies between our initial design assumptions and what might be necessary in order for us to meet our goals for the design of the framework and user interfaces. In addition to user surveys and interviews, we are considering methods for evaluating representations and interfaces that we would use when we are closer to being ready to work directly with potential users of the Framework.

DESIGN OF THE FRAMEWORK IN PROGRESS

The Cambridge Mathematics Framework treats mathematics as a web of ideas with multiple levels of organisation. This web is built as a network in a graph database, in which the mathematical ideas are expressed at nodes and relationships between ideas are expressed as edges. We have developed tools which allow us to search, filter, and visualize the ideas expressed in the Framework, and view different levels and types of information as connected layers (see Figure 1). Currently we are using these tools to design, author, and evaluate the Framework, and in the future they will also form the basis for a set of tools that others will use to interrogate the Framework.

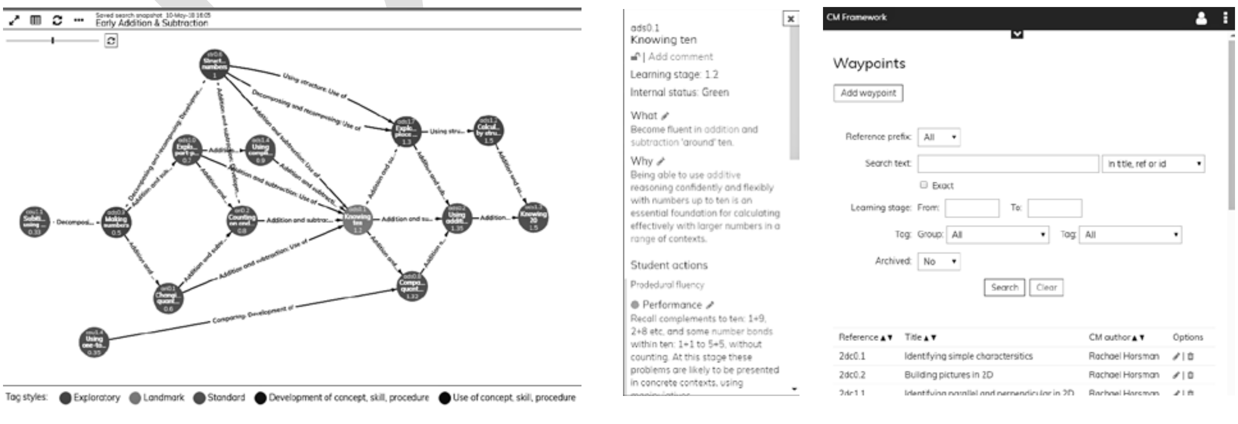

Figure 1: Design tools used to work with and visualise the contents of the Framework

The *mathematical ideas* layer is where we describe mathematical ideas and relationships. The nodes in this layer are *waypoints*, defined as ‘places where learners acquire knowledge, familiarity or

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expertise'. This definition is influenced by characterisation of learning sequences by Michener (Michener, 1978) and Swan (Swan, 2014, 2015). Each waypoint contains a summary of the mathematical idea (the 'what') and why it is included (the 'why'), and lists examples of 'student actions' that would give students the opportunity to experience the mathematics in meaningful ways. All waypoints in the Framework have the above characteristics, but there are also two special cases. *Exploratory waypoints* usually come at the beginning of a set of linked waypoints. At *landmark waypoints*, ideas are brought together such that the whole experience may seem greater than the sum of its parts. We refer to specific waypoints as *standard waypoints* if we need to distinguish them from exploratory or landmark waypoints. Relationships (edges) between waypoints are *themes*, named according to the concept/skill/procedure we believe the relationship to represent (e.g. 3D Shapes, Inference, etc.). The connection between the waypoints is either described as the *development of* a concept/skill/procedure or as the *use of* a concept/skill/procedure.

The Framework is a construct built by individual authors, and so their decisions about themes and waypoints determine which mathematical ideas are expressed in the Framework and how. The tools we use allow us to connect mathematical ideas in multiple ways and to focus on different sets of ideas and connections at different times. Others might make different choices that could still be entirely reasonable representations of a set of mathematical ideas. This is why we write short white papers which we call *research summaries* to explain specific decisions about the creation and structuring of individual themes. The *research* layer contains these research summaries, along with research nodes and edges, all of which are linked to corresponding features in the Mathematical Ideas layer. We are also developing a Glossary layer, which contains glossary nodes in which key mathematical terms or phrases are defined. These are also linked to the appropriate features in the Mathematical Ideas layer. Ultimately, we expect to create additional layers with features which will contribute to task design, professional development and assessment uses of the Framework.

DISCUSSION AND NEXT STEPS

The influence that research can have on curriculum design and development depends in part on the meaning of that research in the work of various stakeholders in curriculum design, and the ways in which different stakeholder groups are able to coordinate their decision-making and actions (whether informed by research or otherwise). In order to evaluate whether the Cambridge Mathematics Framework shows promise in terms of making a positive contribution to this coordination, we will continue our current and planned evaluation efforts, expanding the process of face validation of the contents and the structure of the Framework beyond our core group of collaborators to a broader range of representatives of stakeholder groups. In addition, we are working with collaborators from curriculum design and resource design stakeholder categories on several small pilot projects in order to develop scenarios for use, and we hope to be able to disseminate the results of these in the coming year. In order for the Framework to help designers to have new insights and develop new solutions, so that they can put the raw material for reform into action, we also work according to our knowledge of the context for reform. We continue to deepen and inform our perspectives on the processes and agents of reform, the dynamics between different stakeholder roles, and issues of communication between immediate stakeholders and the public. While our project is focused on the influence of research in curriculum design and development, the other questions in Theme E are considerations which are equally essential to the eventual impact of the Framework in curriculum reform.

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